Rebecca Wenning
Intense Study of the Buoyant Force
Physics – Period 4
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Purpose: The purpose of this lab is through numerous individual experiments to determine how buoyancy (what at this point shall be called the "capacity for an object to sink or float") is measured, what factors affect buoyancy, and, ultimately, to be able to explain the source of the buoyant force.

Materials: Materials will be listed under the individual experiment sections.

Part I: What factors influence buoyancy?

Purpose: The purpose of this experiment is to determine what physical characteristics of various objects cause them to sink, float, or remain neutral in water.

Hypothesis: Based on every-day observation of the phenomenon, it is predicted that only less-dense-than-water objects will float. This is further reasoned by the evidence that hot air rises. Because the atoms move so much more quickly, spread out, and in turn become less dense than the cold air surrounding it, it rises.

Materials:
Either a hand-held weight scale or a triple-beam scale with the plate removed
A large beaker or tub of water
Objects of different masses, shapes, volumes, densities, compositions etc. (To be certain in having only one variable for each experiment, it is required to have at least two objects for each experiment that are identical in everything except for one aspect, such as two objects of the same volume but different mass. For an object identical in all aspects except for shape, it is recommended to use a ball of clay.) (For simplicity of the experiment, only sinking objects will be used.)

Procedure:

1. Reasoning from life experiences, it is concluded that objects weigh less in water than in air. This conclusion is supported after having taken an object, finding its weight in air, and finding its weight in water to be less. However, a way to measure this apparent "change" in weight must be determined before one can proceed with the experiment.

2. By drawing out a vector diagram, it is concluded that while in mid-water, there must be a normal force acting against the object's weight, attempting to equalize the forces. Considering that in order for any object to float, this normal force must be greater than its weight, some unknown force is present. It can then be determined that the greater this unknown force compared to the object's weight, the more easily the object is able to float.

3. It is determined that buoyancy will be defined as the weight of the object in air minus the weight of the object in the water, resulting in a net force of which the source is not yet known.

4. A list of possible determining factors that influence buoyancy is created. The list should include at least the following: mass, volume, shape, surface area, and density.

5. Using the numerous provided objects, small experiments are performed to test the validity of the effectiveness of the factors in the list on their compared buoyant forces. This will be executed by choosing one factor at a time to test, measuring the weights of the compared objects in air and again while being held neutral in the water (not letting the objects touch the bottom of the water-filled beaker or tub.)

Conclusion: It is concluded that only volume has a direct affect on the buoyancy of an object.

Part II: How do the volume of an object and its buoyancy relate?

Purpose: The purpose of this experiment is to use the conclusion from the previous experiment – that volume has a direct affect on the buoyancy of an object – and to find the relationship between object buoyancy and object volume, thereby supporting the conclusion of Part 1.

Hypothesis: Reasoning from support in the previous experiment that volume affects buoyancy, it is merely hypothesized that object volume and buoyancy share some relationship.

Materials:

Either a hand-held weight scale or a triple-beam scale with the plate removed
A large beaker or tub of water
A number of objects varying in volume

Procedure:

1. The volumes of the various objects to be used are calculated.

2. The weight in air and weight in water of each of the objects is found.

3. The weight in water is subtracted from the weight in air for each object, thereby calculating the buoyant force for each object.

4. The buoyant force (y – axis) and volume (x –axis) of each object is plotted on a graph using a computer program and printed out. (See Graph #1 – B_f vs. V)

Conclusion: Using an automatic curve fit, it is determined that the relationship between the buoyancy and volume of an object is linear, resulting in the following equation for this specific system:

Buoyant Force = (9.70e+3) Volume of the object

Part III: How can the density of the liquid affect the buoyancy of an object?

Purpose: The purpose of this experiment is to now turn to how the physical characteristics of the surrounding liquid can affect the buoyancy of an object.

Hypothesis: Seeing how in the previous experiment a constant was included in the final equation relating buoyancy and volume (the constant 9.70e+3), it is hypothesized that that constant was specifically for the system made of water. This constant will change based upon the liquid, changing the buoyancy of an object of a known volume depending upon what compound is surrounding it.

Materials:

Either a hand-held weight scale or a triple-beam scale with the plate removed
A large beaker or tub
Enough of the following solutions to each fill the beaker or tub sufficiently: tap water, salt water, ethyl alcohol, oil, and honey
Any object used to find its buoyancy

Procedure:

1. Find the weight of the object in air

2. For each of the liquids, in turn, find their densities by measuring in a beaker a known volume and finding that volume's mass. The density is calculated by dividing the mass by the volume.

3. For each of the liquids, the object is placed in them and the object's weight measured.

4. The objects' weight in each of the liquids is subtracted from its weight in air, resulting in its buoyant forces for each of the systems.

5. The data for the liquid densities (x – axis) and the object buoyancy for each of the liquid densities (y – axis) are plotted on a graph using a computer program and printed out. (See graph #2 – B_f vs. Density L).

Conclusion: Using an automatic curve fit, it is determined that the relationship between the buoyancy of an object and the surrounding liquid's density is linear. Because this is a direct relationship, the following general statement can be made by referring to the equation derived from the relationship in the previous experiment:

Buoyant Force = (9.70e+3) Volume of the object

$$B_f \quad \rho_\ell V_o$$

The Buoyant force on an object is directly proportional to the density of the liquid times the volume of the object. The second equation cannot be equal merely because the numbers do not work out properly. However, this means there must be a constant. Using the first equation from the last experiment as an example, one can state the following:

$$\frac{B_f}{\rho_\ell V_o} = k$$

To find this constant more conveniently, the data for water found earlier in the experiment can be used as follows: (Note: it is VERY important to have consistent units!)

Buoyant Force of water on object: 0.27 N
Density of water: 955 kg/m^3
Volume of object: 2.79e-5 m^3

$$\frac{0.27\ \text{N}}{(955\ \text{kg/m}^3)(2.79\text{e-}5\ \text{m}^3)} = \text{constant}$$

10m/s^2 = constant

Ironically enough, the constant is equal to the acceleration due to gravity. Therefore the equation can be revised as follows:

$$B_f = \rho_\ell V_o g$$

Part IV: Application of knowledge – for a floating object, how does the volume of the object in a liquid of a specific density compare to the volume of the object out of the liquid of a specific density?

Purpose: The purpose of this section of the lab is to apply mathematically some of the knowledge gained from the previous experiments by predicting how deep a floating object will be when placed in a liquid, and how it compares to specific variables.

Hypothesis: It is hypothesized that the greater the buoyant force of an object of low density object in a high density liquid, the less volume of the object will be in the water, and vice-versa.

Materials:

Paper
Writing utensil

Procedure:

$$B_f = mg$$

1. For the object to be stationary and not accelerating, the buoyant force must be equal to the weight of the object, therefore:

2. From the previous experiment one can state the following:

$$B_f = \rho_l V_{o(in)} g$$

3. These equations can be combined and simplified:

$$mg = \rho_l V_{oin} g$$

$$m = \rho_l V_{o(in)}$$

4. Because of the definition of density:

$$m = \rho_l V_{o(t)}$$

5. These latter two equations can be combined:

$$\rho_l V_{o(in)} = \rho_o V_{o(t)}$$

6. Therefore the relationship can be defined as:

$$\frac{V_{o(in)}}{V_{o(t)}} = \frac{\rho_o}{\rho_l}$$

Conclusion: The conclusion can then be made that should the densities of the liquid and the object be the same, neutral buoyancy occurs. Also, if the densities of the liquid and the object are known, as well as the volume of the object, the percentage of the object volume in the liquid can readily be determined.

Part V: Where does the buoyant force come from?

Purpose: The purpose of this part of the lab is to mathematically determine where the mysterious net force that creates buoyancy on objects comes from.

Hypothesis: Density is indirectly a reason for buoyancy, but what the source of the force is not yet known.

Materials:

Paper
Writing utensil

Procedure:

1. Using substitution of known variables, one can simplify the definition of pressure, P, using a mental model of an area, A, in a column of liquid.

$$P = F/A$$

$$P = W_l/A$$

$$P = mg/A$$

$$P = \rho_l V_l g/A$$

$$P = \rho_l Adg/A$$

$$P = \text{density x depth x g}$$

2. If one compares the P of the top of an object submerged in a solution compared to the P on the bottom of the object, one will notice a net P on the bottom of the object.

3. To simulate such a situation, one can imagine a cube with edge measurements each x submersed in a liquid. The distance from the top side to the top of the water is d_1 and the distance from the bottom side of the cube to the top of the water is d_2. See figure #1.

4. The following relationships have already been proven and can help to prove that the difference in pressures in a liquid column are the source of the buoyant force:

$$B_f = \rho_l V_o g$$

$$F_1 = PA = \rho_l g d_1 A$$

$$F_2 = PA = \rho_l g d_2 A$$

5. For a general situation, to prove the difference in pressure is the source of the buoyant force, the following must be proven:

$$F_2 - F_1 = B_f$$

6. By substitution one can find the following:

$$B_f = (\frac{m_l}{d_1 x^2})(x^3)g = \frac{x}{d_1} m_l g$$

$$F_1 = (\frac{m_l}{d_1 x^2}) g d_1 x^2 = m_l g$$

$$F_2 = (\frac{m_l}{d_1 x^2}) g (d_1 + x)(x^2) = \frac{m_l g d_1 + m_l g x}{d_1}$$

7. To finish the prove, it must be found that $F_2 - F_1 = B_f$,

$$\frac{m_l g d_1 + m_l g d_2}{d_1} - mg = \frac{x}{d_1} m_l g$$

$$\frac{m_l g d_1 + m_l g x - m_1 g d_1}{d_1} = \frac{x}{d_1} m_l g$$

$$\frac{m_l g x}{d_1} = \frac{x}{d_1} m_l g$$

$$\frac{x}{d_1} m_l g = \frac{x}{d_1} m_l g$$

8. Certainly, there is also a more elegant proof:

$$B_f = F_2 - F_1$$

$$B_f = \rho_l g d_2 x^2 - \rho_l g d_1 x^2$$

$$B_f = \rho_l g x^2 (d_2 - d_1)$$

$$B_f = \rho_l g x^3$$

$$B_f = \rho_l g V_o$$

Conclusion: It can be concluded that the buoyant force is due to the net difference between the pressures on the top and bottom of an object because mathematically it has been proven that the difference of the two forces is equal to the buoyant force.

Conclusion to the Lab:

From these numerous activities scrutinizing the buoyant force, the lab participant moves from curiosity, to belief, to pure understanding of a subject in science. The first part of the lab dealt merely with brainstorming what physical factors of the objects might influence their buoyancy. The second part of the lab dealt with the one factor that appeared to be directly related to buoyancy -- object volume. After the graph from the second part of the lab produced in the linear equation a certain constant, the third part of the lab tied in the fact that perhaps liquid densities affect buoyancy. The conclusion of this part of the lab led to the realization that not only is this liquid density – buoyancy relationship linear, but that the unknown constant was in itself the acceleration due to gravity. Part four applied some of the equations determined by the previously determined relationships to derive another relationship relating liquid and object densities to the volume of the object out of the water. Finally, the fifth part of the lab served to prove the source of the mysterious net force found in the liquids – the net difference in pressures above and below the submerged objects.