Motion II

Momentum and Energy

Momentum

- Obviously there is a big difference between a truck moving 100 mi/hr and a baseball moving 100 mi/hr.
- We want a way to quantify this.
- Newton called it Quantity of Motion. We call it Momentum.

Audio Link

Definition: Momentum = (mass)x(velocity)

$$p = mv$$

Note: momentum is a vector.

Massive objects can have a large momentum even if they are not moving fast.

Modify Newton's 2nd Law

$$F = \frac{\Delta p}{\Delta t}$$

- Notes: Sometimes we write Δt instead of just t. They both represent the time for whatever change occurs in the numerator.

- Since $p=mv$, we have $\Delta p=\Delta(mv)$
- If m is constant, (often true) $\Delta p=m\Delta v$

$$F=\frac{\Delta p}{\Delta t}=\frac{m\Delta v}{\Delta t}=ma$$

- Impulse is defined as

$$\Delta p=F\Delta t$$

- Impulse looks like the strength of an interaction over time

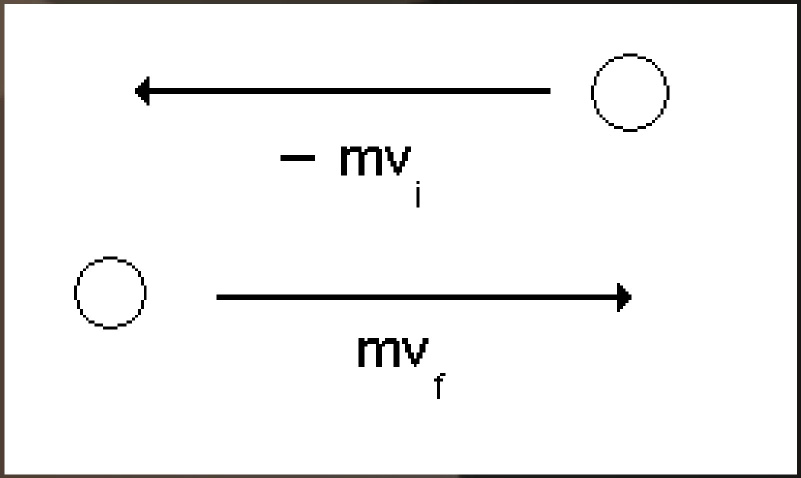
Examples

- Baseball
- $\Delta p = p_f - p_i$

  $= mv_f - (-mv_i)$

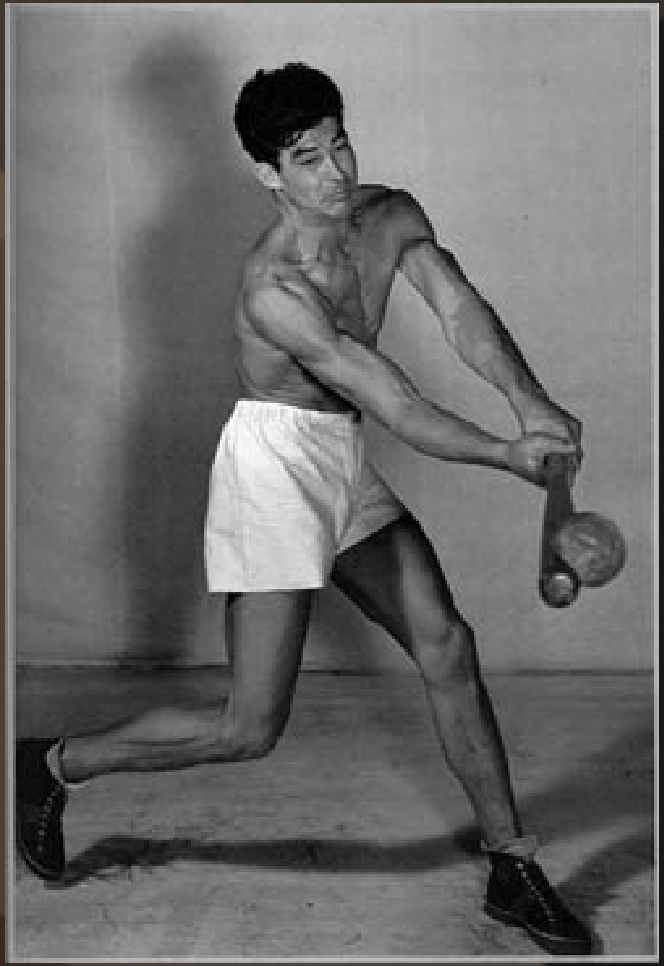
  $= m(v_f + v_i)$

$F = \Delta p / \Delta t$

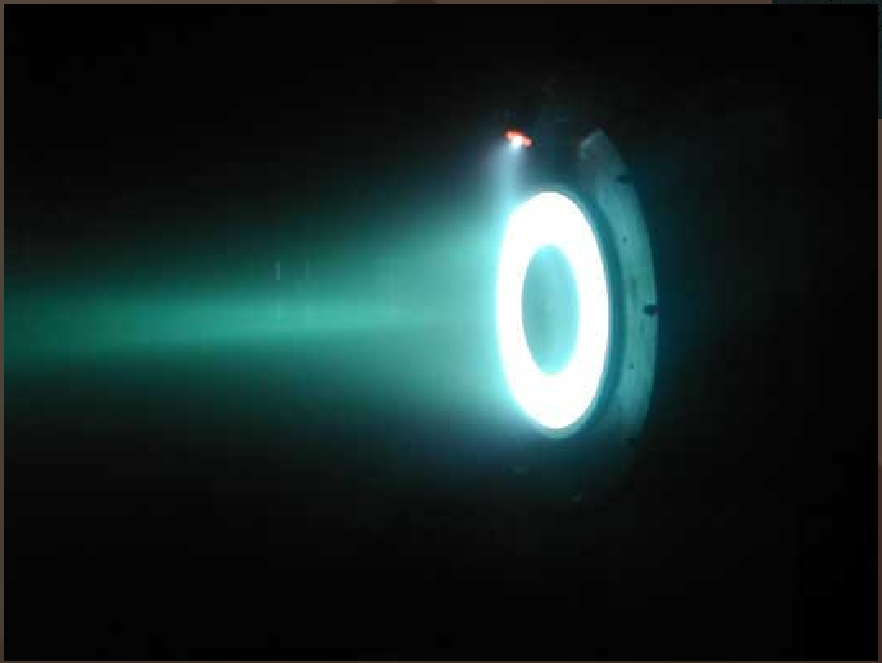


Δt is the time the bat is in contact with the ball

- Realistic numbers
- M=0.15 kg
- V_i = -40 m/s
- V_f = 50 m/s
- Δt = (1/2000) s
- F=27000 N
- $a=F/m$=180000 m/s^2
- a=18367 g
- slow motion photography



- Ion Propulsion Engine: very weak force (up to about a Newton) but for very long times (months)



- Crumple Zones on cars allow for a long time for the car to stop. This greatly reduces the average force during accident.

- Landing after jumping

A woman has survived after falling from the 23rd floor of a hotel in the Argentine capital, Buenos Aires.

Her fall was broken by a taxi, whose driver got out moments before the impact crushed the roof and shattered the windscreen. Eyewitness said the woman had climbed over a safety barrier and leapt from a restaurant at the top of the Hotel Crowne Plaza Panamericano. She was taken to intensive care for treatment for multiple injuries. The woman, who has not been named, is reported to be an Argentine in her 30s. The taxi driver, named by local media as Miguel, said he got out of his vehicle just before the impact after noticing a policeman looking up. "I got out of the car a second before. If I had not got out, I would have been killed," he told Radio 10. "I was only 10 metres from the impact. It made a terrible noise," he added. The Hotel Crowne Plaza Panamericano overlooks the Obelisk, one of the best known landmarks in Buenos Aires.

- Estimate *m*=50kg
- *h* = 23 x 3.5m = 80.5m (distance)

$$h=\frac{1}{2}at^2$$

$$t=\left(\frac{2h}{a}\right)^{1/2}$$

$$t=\left(\frac{(2)(80.5\text{m})}{9.8\text{m}/s^2}\right)^{1/2}=4.05\,s$$

$$v=at=39.7\text{m}/s=89\,mph$$

Estimate Acceleration on Impact

$$a = \frac{\Delta v}{\Delta t} = \frac{\Delta v}{\Delta x}\frac{\Delta x}{\Delta t} = \frac{\Delta v}{\Delta x} v_{ave}$$

$$\frac{\Delta x}{\Delta t} = \frac{1}{2}\left(v_f - v_i\right) = \frac{1}{2}\Delta v = v_{ave}$$

$$a = \frac{v_i v_i / 2}{\Delta x}$$

Notice that the velocity is directed down and is really negative or zero.

- Case 1: stops in deformation distance of body ~3cm.

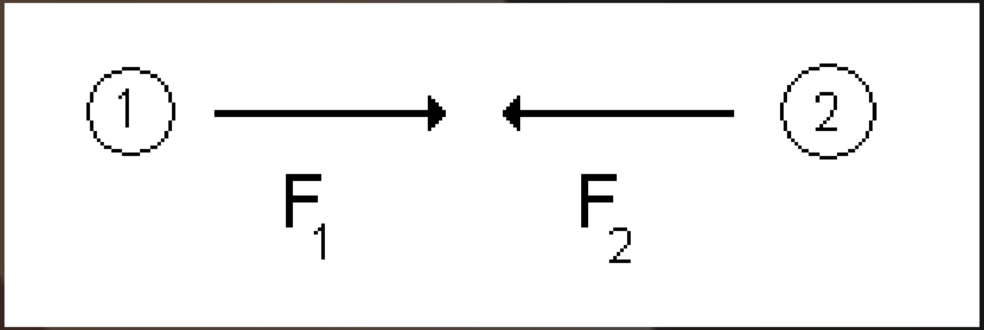
$$a=\frac{\left(39.7\mathrm{m}/s\right)^2/2}{0.03\,m}=26268\,m/s^2$$

- Case 2: stops in height of car, guess ~1 m.

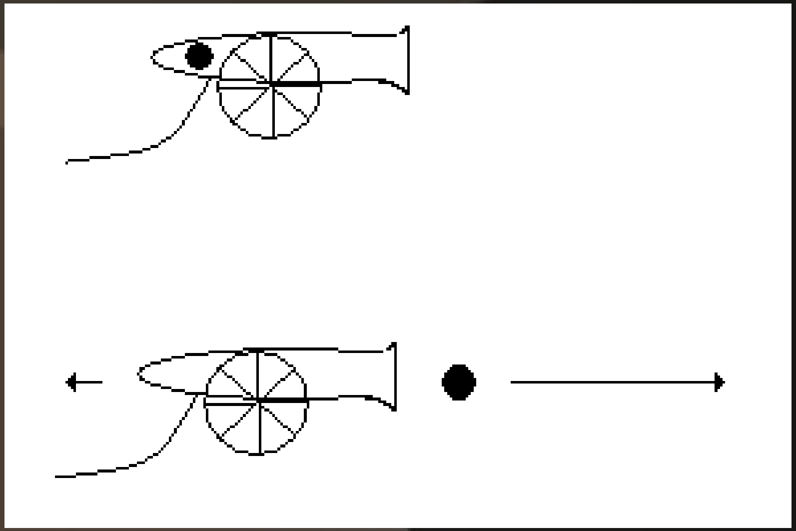
$$a=\frac{\left(39.7\mathrm{m}/s\right)^2/2}{1\mathrm{m}}=788\,m/s^2$$

Conservation of Momentum

- Newton's 3rd Law requires $F_1 = -F_2$
- $\Delta p_1/\Delta t = -\Delta p_2/\Delta t$
- Or
- $\Delta p_1/\Delta t + \Delta p_2/\Delta t = 0$
- Or
- $\Delta p_1 + \Delta p_2 = 0$
- NO NET CHANGE IN MOMENTUM



Example: Cannon



- M_c = 1000kg cannon
- M_b = 5kg cannon ball

Before it fires

$$p_c + p_b = 0$$

Must also be true after it fires.

Assume V_b=400 m/s

$$M_c V_c + M_b V_b = 0$$

$$V_c = -M_b V_b / M_c$$

$$= -(5\text{kg})(400\text{m/s})/(1000\text{kg})$$

$$= -2\text{m/s}$$

Conservation of Momentum

- Head on collision
- Football
- Train crash
- Wagon rocket
- Cannon

ENERGY & WORK

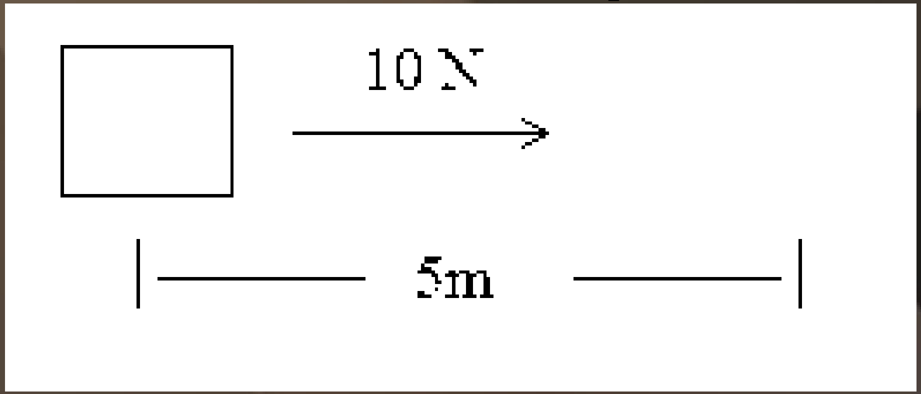
- In the cannon example, both the cannon and the cannon ball have the same momentum (equal but opposite).
- Question: Why does a cannon ball do so much more damage than the cannon?
- Answer: ENERGY

WORK

- We will begin by considering the “Physics” definition of work.
- *Work = (Force) x (distance traveled in direction of force)*
- $W = F \times d$
- Note: 1) if object does not move, NO WORK is done.
- 2) if direction of motion is perpendicular to force, NO WORK is done (important for circular motion)

Audio Link

Example



- Push a block for 5 m using a 10 N force.
- Work = (10N) x (5m)

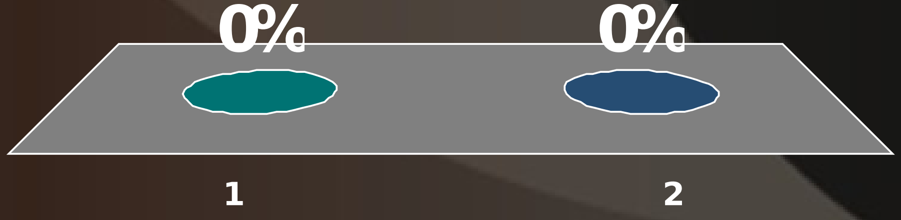
  = 50 Nm

  = 50 kgm^2/s^2

  = 50 J (Joules)

I have not done any work if I push against a wall all day but it does not move.

1. True
2. False



- ENERGY is the capacity to do work.
- The release of energy does work, and doing work on something increases its energy

$$E = W = F \times d$$

- Two basics type of energy: Kinetic and Potential.

Kinetic Energy: Energy of Motion

- Heuristic Derivation: Throw a ball of mass *m* with a constant force.

$$KE = W = F \times d = (ma)d$$

But $d = \frac{1}{2} at^2$

$$KE = (ma)(\tfrac{1}{2} at^2) = \tfrac{1}{2} m(at)^2$$

But $v = at$

$$KE = \tfrac{1}{2} mv^2$$

Potential Energy

- Lift an object through a height, *h*.

$$PE = W = F \times d = (mg)h$$

$$PE = mgh$$

Note: We need to decide where to set h=0.

Units of energy and work

- W=Fd $$Nm=\frac{kgm}{s^2}m=\frac{kgm^2}{s^2}=J$$
- KE=$mv^2/2$ $$kg\left(\frac{m}{s}\right)^2=\frac{kgm^2}{s^2}=J$$
- PE=mgh $$kg\frac{m}{s^2}m=\frac{kgm^2}{s^2}=J$$

Other types of energy

- Electrical
- Chemical
- Thermal
- Nuclear

Conservation of Energy

- We may convert energy from one type to another, but we can never destroy it.
- We will look at some examples using only PE and KE.

$$KE + PE = const.$$

OR

$$KE_i + PE_i = KE_f + PE_f$$

Drop a rock of 100m cliff

- Call bottom of cliff h=0
- Initial KE = 0; Initial PE = mgh
- Final PE = 0
- Use conservation of energy to find final velocity

$$mgh = \frac{1}{2}mv^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

- For our 100 m high cliff

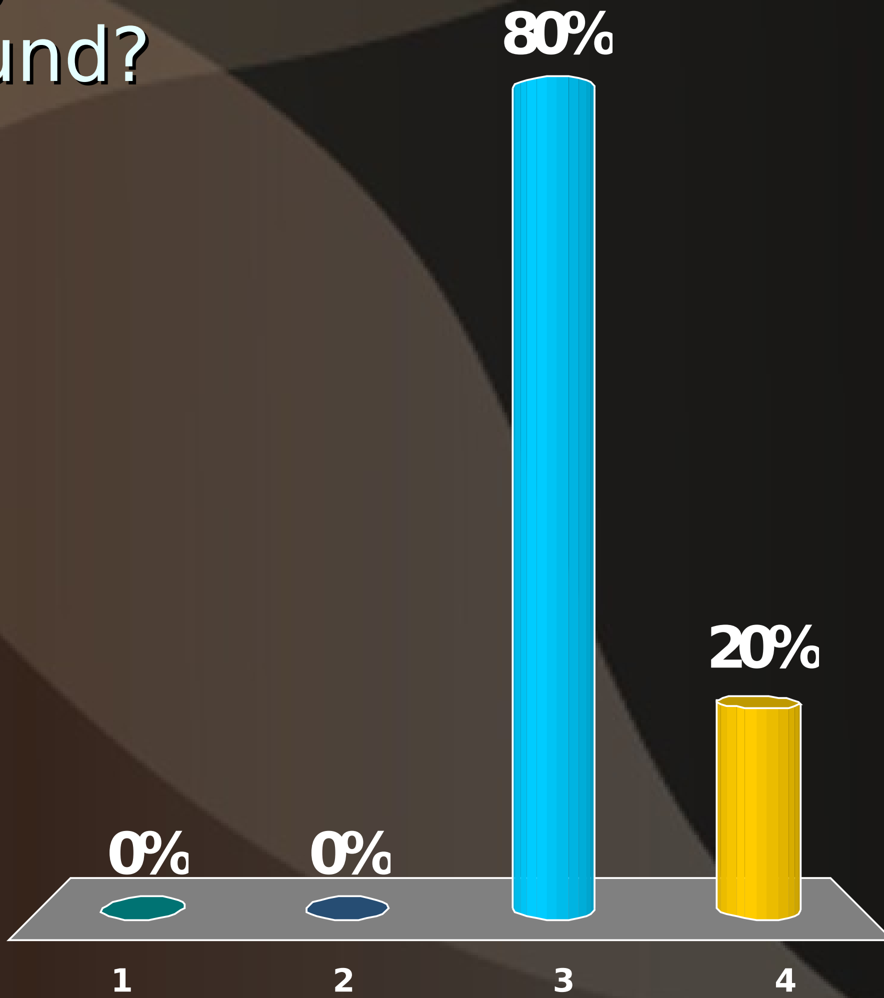
$$v = \sqrt{2\left(9.8 m/s^2\right)\left(100\, m\right)}$$

$$v = 44.3 m/s$$

- Note that this is the same as we got from using kinematics with constant acceleration

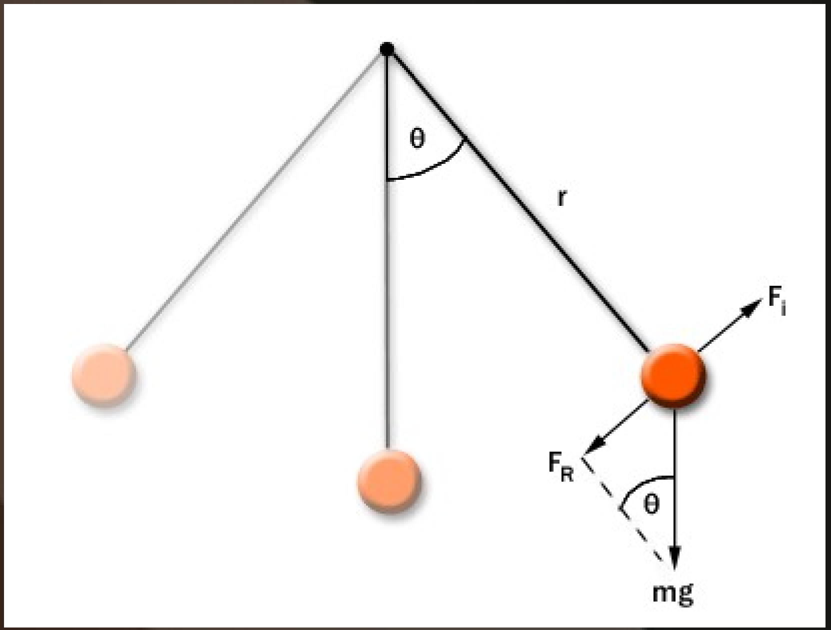
A 5kg ball is dropped from the top of the Willis (Sears) tower in Downtown Chicago. Assume that the building is about 300 m tall and that the ball starts from rest at the top. How fast is it moving when it reaches the ground?

1. 5880 m/s
2. 2940 m/s
3. 76.7 m/s
4. 54.2 m/s



Pendulum

- At bottom, all energy is kinetic.
- At top, all energy is potential.
- In between, there is a mix of both potential and kinetic energy.
- Bowling Ball Video



Energy Content of a Big Mac

- 540 Cal
- But 1Cal=4186 Joules

$$540\,Cal\left(\frac{4186\,J}{1\mathrm{C}}\right)=2{,}260{,}440\,J$$

How high can we lift a 1kg object with the energy content of 1 Big Mac?

- PE=mgh
- h=PE/(mg)
- PE=2,260,440 J
- m=1 kg
- g=9.8 m/s

$$h=\frac{2,260,440\,J}{(1\,kg)(9.8\mathrm{m}/s^2)}=230,657\,m=230.7\,km=143\,mi$$

- 2 Big Macs have enough energy content to lift a 1 kg object up to the orbital height of the International Space Station.
- 150 Big Macs could lift a person to the same height.
- It would take about 4,000,000 Big Macs to get the space shuttle to the ISS height.
- Note: McDonald’s sells approximately 550,000,000 Big Macs per year.

Revisit: Cannon Momentum

- 1000kg cannon
- 5kg cannon ball

Before it fires

$P_c + P_b = 0$

Must also be true after it fire

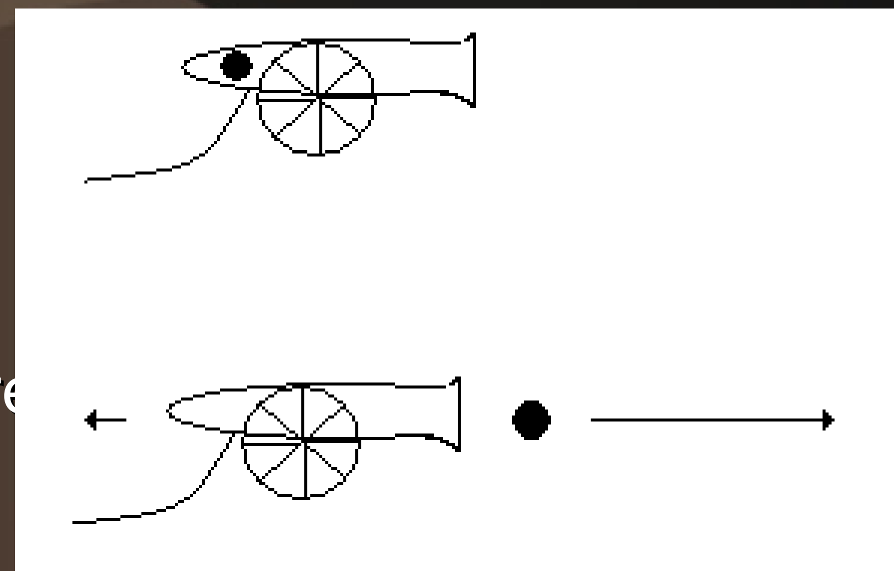
Assume V_b=400 m/s

$M_cV_c + M_bV_b=0$

$V_c=-M_bV_b/M_c$

$= -(5kg)(400m/s)/(1000kg)$

$=-2m/s$



Cannon:Energy

$KE_b = \frac{1}{2} mv^2 = 0.5\ (5kg)(400m/s)^2$

$= 400{,}000\ kgm^2/s^2$

$= 400{,}000\ J$

$KE_c = \frac{1}{2} mv^2 = 0.5\ (1000kg)(2m/s)^2$

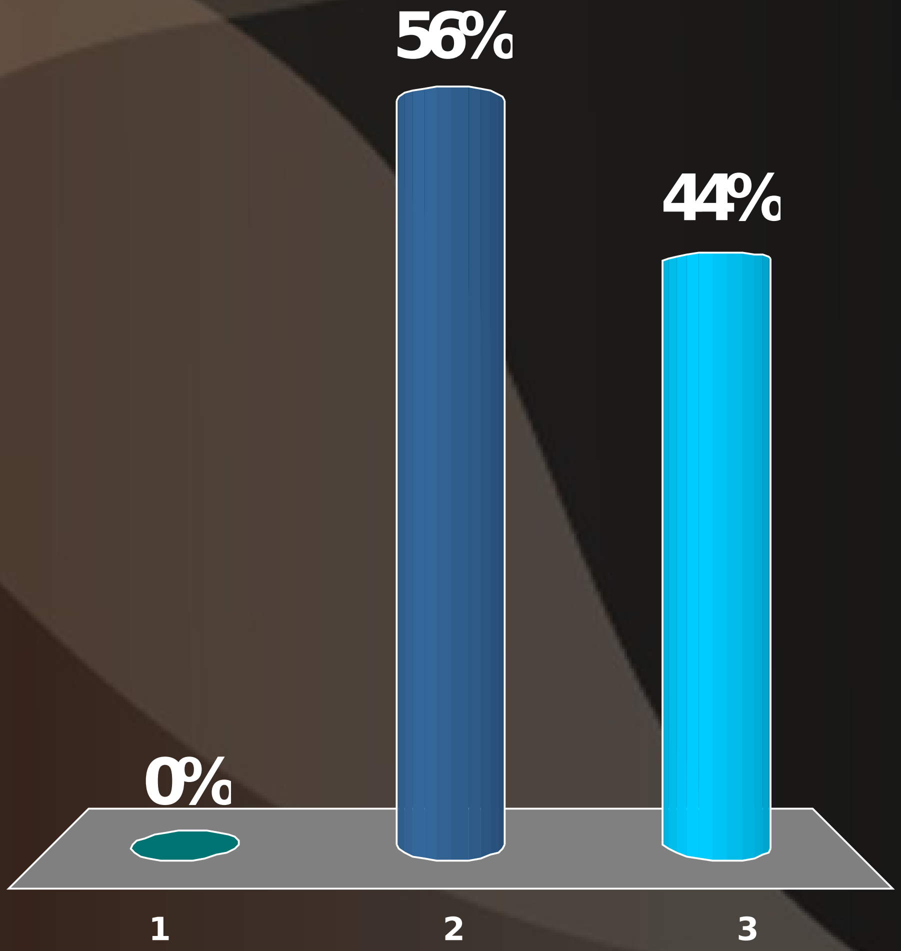
$= 2{,}000\ kgm^2/s^2$

$= 2{,}000\ J$

Note: Cannon Ball has 200 times more ENERGY

When I drop a rock, it start off with PE which gets converted in to KE as it falls. At the end, it is not moving, so it has neither PE or KE. What happened to the energy?

1. Energy is not conserved in this case.
2. The energy is converted into Thermal energy
3. The energy is transferred to whatever it is dropped on.,



Important Equations

$$v=\frac{\Delta d}{\Delta t}$$

$$F=ma=\frac{\Delta p}{\Delta t}$$

$$p=mv$$

$$a=\frac{\Delta v}{\Delta t}$$

$$KE=\frac{1}{2}mv^2$$

$$I=\Delta p$$

$$d=\frac{1}{2}at^2$$

$$PE=mgh$$