A Multiple Case Study of Novice and Expert Problem Solving in Kinematics with Implications for Physics Teacher Preparation

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In recent years, physics education researchers and cognitive psychologists have turned their attention to the question of how individuals solve basic physics problems. The author summarizes the surprising results of a multiple case study in which three experts and three novices were observed as they solved kinematics problems using a "think aloud" protocol. Follow-up interviews and content analysis led the researcher to conclude that expert problem solvers do not always follow the most efficient routines, nor do they always use the most effective methods for teaching basic problem-solving skills to students. These circumstances have important implications for physics teacher education.

The class began on time at 10:25 AM with the instructor asking the students if they had any questions about the kinematics homework problems that they were supposed to have attempted the night before. A discussion dealing with three homework problems (and one example problem) ensued for the next 45 minutes. While the instructor was solving these problem, students observed intently. The majority of the students listened while the instructor talked and worked on the board, yet about one third of the students ceaselessly recorded in notebooks everything that the instructor wrote.

In each case of problem solving, the treatment by the instructor was consistent and methodical. The instructor began with a statement of the problem. Next, he drew a picture. Thirdly, he stated what was known or given as part of the problem. Fourth, he identified a principle by which the problem could be solved. Fifth, he stated the relevant equation that related the knowns and unknowns. Sixth he restated the knowns and unknowns. He then solved the equation for the required unknown, inserted the knowns, and carried out the arithmetic calculation. The instructor then made reference to checking the answer for reasonableness. The instructor’s approach to problem seemed clear and, yet, something seemed to be missing. During the problem-solving session there were 19 questions asked by students. The questions, interestingly enough, were more frequently metacognitive questions (“How do you know when to...?” and “What do you do if...?” and “How do you go about...?”) than any other variety.

Beginning at 11:10 AM, the instructor moved on to a 20-minute lecture about Newton's first and second laws. He did not provide many significant real-life examples of the first law, and the second law was treated entirely at a theoretical level. During this time, all students appeared to be diligently taking notes. At the outset of the lecture portion of the class, the instructor dealt momentarily with the alternative conception that moving things need a constant force to keep them in motion.

At the end of this session, and near the end of the class, the instructor worked another example problem. He assigned 16 exercises for homework at the end of the hour. Eight of the exercises were questions, six were “standard” problems, and two were “challenge” problems. The students diligently recorded the list of required homework problems and promptly left the classroom at the end of the period. Another typical introductory physics class had come and gone.

What do we leave students with at the end of a series of such introductory physics lessons? Are students better able to solve physics problems now that they have seen a few examples? Do they have a metacognitive understanding of this simple problem-solving process that is so frequently tendered with almost every lecture-based recitation class in which problem solving is addressed? Do courses that have as their greatest emphasis the solution of textbook problems leave the students with the perception that the scientific process is little more than searching for the right equation? How important are concrete examples to true student understanding of physical phenomena? These are only a few of the questions that might arise from intently watching and seriously reflecting on what happens in many introductory physics classes. To focus on all these questions would be too great a task in the limited space available for this article and, so, a more narrow view will be centered on the difficulties associated with teaching the general problem-solving paradigm so frequently taught in didactic introductory-level physics courses -- find the knowns and unknowns, state the relationship between them, and solve for the unknown.

Problem Solving in Physics

In recent years physics education researchers and cognitive psychologists have turned their attention to the question of how individuals solve physics problems. Recent research has focused on two areas as they pertain to physics problem solving: (a) the overall plan of attack used to solve problems, and (b) the identification and use of heuristics in problem solving. The researchers generally approach a study of the first focus area by comparing and contrasting the performance of novices (generally defined to be students in introductory physics classes) with that of experts (generally defined to be physics teachers). Studies in the area of problem solving frequently utilize qualitative approaches and involve a relatively small number of subjects. "Think-aloud" protocols are normally used in these efforts. Computer models are generally associated with the heuristic aspect of problem solving and will not be dealt with in this article.

A clear and concise definition of problem solving must be given if the problem statement is to be meaningful. A review of secondary sources shows that there are a number of definitions of the word "problem," but the definition that is most apropos to this project is a characterization -- work associated with those tasks found at the end of chapters of introductory physics text books. Typically, these tasks involve a statement of information and/or circumstances, and an additional variable or variables are determined on the basis of the information provided. These tasks tend to be very specific and the work and goal well defined. Problem solving then is the process of attaining the goal of any specified problem.

Context

Studies of novice and expert physics problem solvers have suggest that there are two distinct and contrasting patterns of problem solving among experts and novices. These variations have led to the formulation of two major models for problem solving. According to Larkin et al. (1980), expert problem solving is typified by the KD model, the so-called knowledge-development approach. Novice problem solving is typified by the ME model, the so-called means-end approach. In the ME model the student typically works "backward" from the unknown to the given information. Under this scenario the novice problem solver (NPS) essentially writes an equation and then associates each term in the equation with a value from the problem. If there are additional unknowns, the problem solver moves on to the next equation. In the KD model the expert proceeds in the opposite direction, working forward from the given information. Under this second scenario, the expert problem solver (EPS) associates each of the knowns with each term of the equation as the equation is set up. That is, novices move from equations to variables, while the experts move from the variables to the equation.

The research in the area of physics problem solving accelerated rapidly in the early 1980s and is now the focus of attention in the research literature. There are a number of questions left unresolved, including those given by Maloney (1994), "What knowledge do novices typically use when faced with physics problems?" and "How is the knowledge that a novice possesses organized in memory?" and "How do alternative conceptions affect novices' representations?" However important these questions, the basis of this research still depends upon the answer to the question, "*How do problem-solving approaches differ between novice and experts?*"

Method

In case studies, the researcher is the primary research instrument. When this is the case, validity and reliability concerns can arise. The human investigator may misinterpret or hear only certain comments. Guba and Lincoln (1981), as well as Merriam (1991), concede that this is a problem with case study work. Yin (1994, p. 56) lists six attributes that an investigator must possess to minimize problems with validity and reliability associated with the use of the human research instrument.

- A person should be able to ask good questions -- and to interpret the answers.
- A person should be a good "listener" and not be trapped by his or her own ideologies or preconceptions.
- A person should be adaptive and flexible, so that newly encountered situations can be seen as opportunities, not threats.
- A person must have a firm grasp of the issues being studied, whether this is a theoretical or policy orientation, even if in an exploratory mode. Such a grasp focuses the relevant events and information to be sought to manageable proportions.
- A person should be unbiased by preconceived notions, including those derived from theory. Thus a person should be sensitive and responsive to contradictory evidence.

The researcher believes that he exhibited these personal characteristics, though "no devices exist for assessing case study skills." (Yin, 1994, p. 56)

Five kinematics physics problems were written for this project. The five questions ranged from simple one-step problems with a single output variable, to more complex two-step problems where more than one output variable was requested. These problems used in this study can be found in Appendix A.

Three faculty members and four students were then self-selected to participate in this study. All faculty members were male; one of four physics students was female. Though this may at first appear to be too large a sample for a case study, “any finding or conclusion in a case study is likely to be much more convincing and accurate if it is based on several different sources of information.” (Yin, 1994, p. 92) The problem-solving skills of these individuals were examined through observation, interview, and content analysis. Such use of multiple data sources also enhances validity and reliability via triangulation.

All volunteer faculty members participating in this study had experience teaching introductory physics courses for non-majors. All students were volunteers who were currently enrolled in an introductory, algebra-based physics course for non-majors at a middle-sized Midwestern university. Students were informed that a wide range of problem-solving abilities were needed, and that excellence in problem solving was not a prerequisite for participating in the study. (The female student was subsequently dropped from the study due to an apparent lack of ability to solve even rudimentary algebraic equations.)

Three data collection strategies were used in this project. Participants first solved the five physics problems using a "think aloud" protocol. The researcher listened to the problem solvers, recording pertinent details dealing with the solution of the problems. He later coded these comments for analysis. Following problem solving, the researcher collected the written work which would be used in content analysis, and then commenced a semi-structured interview to achieve a greater understanding of the problem-solving process. In follow-up interviews, faculty members were asked three questions common to all study participants, and two additional questions reserved to expert problem solvers. Students were asked the same three common questions and three additional student-specific questions. The questions can be found in Appendix B.

Findings from Observations

Appendix C shows the coding plan for problem solver statements made while working on the problems using a think aloud protocol. The coding plan consists of steps in a theoretical scheme of problem solving enunciated by Heller, Keith, and Anderson (1992), and modified and extended slightly for this study. Each step of the problem-solving process is operationally defined with descriptors. For instance, a problem solver can be said to be visualizing the problem if he or she draws a sketch, identifies the known variables and constraints, restates the question, or identifies the general approach to solving the problem. While problem solvers were working problem number one (and all subsequent problems), the researcher recorded statements for later coding. The results of the coding can be found in Table 1.

Model	EPS #1	EPS #2	EPS #3	NPS #1	NPS #2	NPS #3
1	1	2	2	1	2	1
2	2	3	1	2	1	3
3	3	1	3	3	3	4
4	4	4	4	4	4	6
5	5	5	5		5	5
						7
						5
						3
						7
						3
						4
						5

Table 1. *Logical approaches used by expert and novice problem solvers to solve problem one.*

This table shows the logical approaches used by expert and novice problem solvers. If a problem solver uses what is theoretically the most efficient scheme for solving the problem, then his solution should consist of five sequential steps: 1, 2, 3, 4, and 5. If expert problems solvers (EPS’s) depart substantially and consistently from this model, it might lead the researcher to conclude one of two things: either these particular EPS’s are inefficient, or the model proposed by Heller et al. is simply wrong.

The data tabulated in Table 1 shows that EPS’s do not generally follow the same paths to a solution as the theoretical model. In all three cases, the EPS’s chose different routes to solve the problem. These paths were 123, 231, and 213. Novice problem solvers (NPS’s) #1 and #2 took similar mixed routes, while NPS #3 departed from the general problem solving model when he failed to include step two. Among the six problem solvers, this was the only person to neglect this step, leading possibly to the long, convoluted solution to the problem as indicted by the twelve steps. Interestingly enough, five of the six problems solvers made the effort to mentally check their answers for apparent correctness.

The overall impression gained by the researcher while observing the problem solvers was that the problem-solving procedures utilized by novice problem solvers are very unstructured and inefficient. Problems are not systematically approached, knowns are rarely written down in equation form (for instance, $a = 1$ m/s^2), starting equations are rarely written down, equations are not solved for unknown variables before inserting the knowns, work is done without units, solving algebraic equations appears to be a problem for most, etc. Students, in many cases, quite randomly choose equations to solve for the unknown. They, not infrequently, expected a calculator to “solve” the problem for them. One student in particular regularly multiplied and divided numbers in a random fashion looking for solutions that “looked right.” This procedure might work on a multiple-choice test -- something that is normally used at the introductory level -- but not in this research project where students had to derive precise answers of their own. In general, the time required for EPS’s to solve problems was one third that required by NPSs.

Findings from Interviews

It is clear from the interview process that in the area of kinematics, students tend to follow the same general procedures as the experts when it comes to problem solving: search for knowns and unknowns, establishing or finding a relationship between the knowns and unknowns, and then solve for the unknown. The general procedure for problem solving is shown in Figure 1. In some cases the students would check their answers to see if they made sense; this was normally the case with experts. Checking the answer generally took the form of looking at the magnitude and sign of the solved variable. The students interviewed seemed to be clear on the overall process. When they did have trouble, it was in selecting the appropriate equation to relate the known and unknown variables through the most direct route. In this procedure two faculty members were very efficient; however, one expert problem solver almost invariably started the problem-solving process with the same kinematics equation, no matter what the original given quantities were.

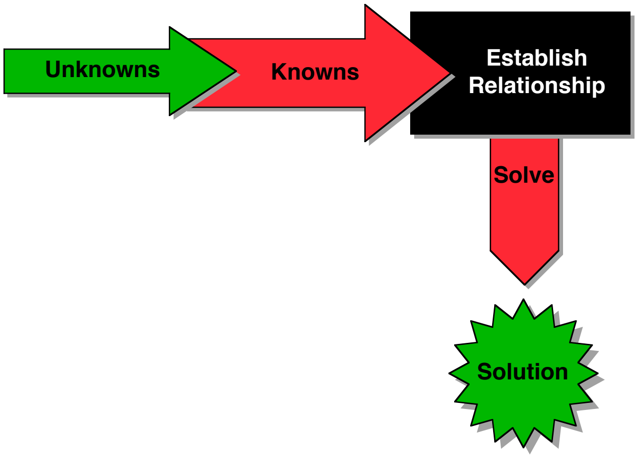

Figure 1. Problem-solving flow-chart. *The general problem-solving procedure appears to consist of identifying the known and unknown variables, finding a mathematical relationship between the variables, and then solving for the unknown. Unfortunately, some students do not appear to have a clear understanding of the thought processes that take place in the black box entitled "Establish Relationship."*

Two students were unable to explain clearly the "black box "procedure for selecting the appropriate kinematics equation to relate the variables (see Figure 1). For instance, "I look to fit all the information into a model" and "I see what formula gives me the information I need." The result of this uncertainty was clearly evident as these two students randomly selected one equation after another in an effort to "plug and chug" their way through the problem set. One student was clear about the procedure, "The equation I would select would be that which has one unknown variable -- the one you are looking for. Alternatively, using a formula with two unknowns where one of the unknowns can be obtained with the use of another formula." All problem solvers, novices and experts alike, appeared to use the means-ends approach to solve the five physics programs provided.

The physics teachers were asked to explain how they taught kinematics problem solving in their introductory courses. In all cases teachers indicated that they made use of examples almost exclusively. In one case, an instructor noted that from time to time he would attempt to clarify the process by explaining the process in words;

in another case an instructor indicated that he would never use a metacognitive approach. In his words, "...I do not discuss general strategies.... I'm not sure some students at this level can conceptualize general strategies. Strategies are drawn by example." Another instructor noted, "I don't think that there is any particular procedure that you can describe to the students for them to become more expert. In special areas I point out what they have to do to recognize the unknown, the data, and what sort of formula for them to use. Students often randomly search for formulas. I warn them against this." In no case was any attempt made to explain explicitly what was going on in the mind of the instructor to explain the equation selection process.

The students interviewed mentioned that they did make use of examples to learn how to do kinematics problem solving. In all three cases the students reported reading over the example, and sometimes working the example, in an effort to comprehend the general procedure. They did not indicate using examples as templates for solving problems except in one instance. This student reportedly resorts to using examples like templates to find one variable in a two-step problem in which the desired variable is not immediately obtainable directly from an equation.

When queried, student expressed the opinion that they had learned general problem-solving strategies *prior* to taking the physics class mentioned in this study. One student attributed his physics problem-solving skill to a high school classmate; another to life experiences; and yet another to related coursework in business classes. Students generally felt that their problem-solving skills were enhanced by taking the physics course, and this helped them to gain a broader perspective on the problem-solving process. There was a general consensus that the instructors did very little to help students learn the fundamental intellectual processes of mathematical problem solving in physics.

Findings from Content Analysis

Subsequent to the follow-up interviews, the written work of problem solving was collected for content analysis. The procedures used by problem solvers were coded on the basis of equations used to find intermediate or final unknowns following the work of Simon and Simon (1978). The equations referred to are those appearing on the problem sheet shown in Appendix A. The first equation is labeled 1, the second 4, the third 5, the fourth 7, and the fifth 8. This numbering sequence was chosen to remain consistent with previous research on kinematics problem solving. The coding procedure is "shorthand" that indicates how problem solvers approached problems. For instance, if a problem solver found the average velocity, $\bar{v}$, using equation 5, then the approach was coded ($\bar{v}5$). If the instantaneous velocity, v, was found from equation 5, then the approach was coded ($v5$).

Table 2 shows the results of coding the mathematical steps used by EPS's and NPS's. The designations running horizontally along the top numerically distinguish EPS's and NPS's. The numbers running vertically along the left side of the table indicate problem number. Each cell contains the equation-based problem solving approach. False starts have not been included in this table, nor have unsuccessful attempts to solve problems. If a cell in the table is blank, it is an indication that the problems solver was unable to find the correct solution.

#	EPS #1	EPS #2	EPS #3	NPS #1	NPS #2	NPS #3
1	$\bar{v}5 \square a8 \square t4$	$\bar{v}5 \square a7 \square t4$	$\bar{v}5 \square a8 \square t4$	$\bar{v}5 \square t1$	$\bar{v}5 \square t1$	$\bar{v}5 \square t1$
2	$a4 \square x7$	$a4 \square x7$	$a4 \square x7$			
3	$t4$	$t7$	$t4$		$t4$	$x8 \square t7$
4	$a8$	$at4 \square at4 / t7$	$a8$		$a8$	$a8$
5	$v8 \square t4$	$t7 \square v4$	$v8 \square t4$	$t4$ *	$v8 \square \bar{v}5 \square t1$	$v8 \square t4$

* Did not solve for v.

Table 2. *Mathematical approaches used by expert and novice problem solvers.*

From an inspection of the approaches outlined in this table, it is clear that not all expert problem solvers determine unknowns in the same fashion or with the same efficiency (efficiency being defined as working toward the answer by taking the most direct route -- using the fewest number of steps and equations to solve for an unknown). Admittedly, there are several ways to solve each of these problems, with some routes being different but equally efficient. This can be seen in the solution of problem 5 by expert problem solvers.

Differences in problem-solving efficiencies were notable among EPS's attacking problem 4. For example, compare the procedure of EPS #2 with those used by EPS #1 and EPS #3. EPS #2 used a solution procedure that was less efficient than that used by other EPS's. EPS #2 solved for the product of a and t from equation 4, and then divided this product by $t7$ while the other EPS's solved equation 4 directly. This appears to have do with EPS #2's propensity for beginning most problems with a statement of equation 7, and then searching for variables to insert into the equation -- not always the most efficient procedure.

Interestingly, some NPS's exhibited what appears to be greater insight in solving some problems than EPS's. For instance, note how all NPS's solved problem 1 in a much more direct fashion than any EPS, not solving for acceleration (a) in order to find t. Though the table does not show it, NPSs took a significant number of dead-end approaches to solving the problems.

Discussion

The findings of this research project do not lend support to the claim that expert problem solvers tend to use a KE approach and novice problems solvers an ME approach -- at least in the area of kinematics. Both NPS's and EPS's used the same technique of searching for an equation among a group of equations that contains the end variable. They then worked from this end using any means necessary. One might argue that their is no alternative to the solution of kinematics problems, but the contrasting solution of problem 1 by EPS's and NPS's would seem to indicate that the students interviewed have used a more "insightful" KE approach than did the EPS's.

It appears that the general procedure for solving kinematics problems (find the knowns and unknowns, state the relationship between them, and solve for the unknown) are clear to the students studied. It is also clear that these students have *not* learned detailed problem-solving procedures by watching instructors solve example problems. They seem to have done so on their own – in other courses or through friends. What students are not consistently clear about is how to select the appropriate kinematics equation or equations to relate and solve for the problems' unknown. Evidently some students have been unable to figure out by observation the relatively sophisticated black box mental process the instructor goes through to select the appropriate kinematics equation.

What was not self-evident to the physics instructors is that students would appear, in some cases, not have a good understanding of the equation-selecting process that goes on quickly in instructors' minds. Though instructors argue that students appear to learn from example, one of the most important examples that is lacking is that which illustrates the thinking process that the course instructor goes through to select the appropriate equation among those available in kinematics. In one case a NPS had a clearer view of this than, perhaps, an EPS. This same EPS noted that he didn't think there was a general problem-solving process that students could comprehend. Perhaps this is so because that EPS never established a clear procedure for himself as is evidenced by the rigid, lock-step procedure of attempting to solve the kinematics problems by starting with equation 7 each time.

It is clear from subsequent discussions with each of the faculty members participating in this project that they may well generally lack a clear understanding of students' problem-solving difficulties. They tend to see a host of student problem-solving difficulties such as: (a) failing to use a systematic process to solve problems, (b) failure to identify variables with known quantities, (c) adding dissimilar knows together such as velocity and acceleration, (d) trying to solve equations without writing them down, (e) using calculators to solve the problems rather than the equation for the unknown, (f) randomly selecting equations to be solved for the unknown variable, (g) making algebraic errors, (h) confusing v with $\bar{v}$., (i) failing to recognize simplifying conditions ($v = 0$ at top of flight path for a projectile, for instance), and that (j) novices are much less systematic than experts in both thinking and writing down their work. The instructors studied do not seem to be aware, however, of the difficulties students face when attempting to figure out what is going on in the black box of establishing relationships between variables. How widespread this evident unawareness on behalf of instructors is not known.

Because the faculty members interviewed possibly have never taken the time to analyze student problem-solving difficulties, and then triangulated those observations to lend credibility to their findings, they seem not to be aware of the central issue of problem solving by NPS's. Additionally, if the instructors studied were to more closely examine the nature of the questions that so many students ask during class, they might be more aware of the need for students to have a metacognitive understanding of the problem-solving process being used, and particularly those occurring in the dark recesses of the black box known as "establish relationship."

Two questions that arose in the mind of the interviewer as he talked with students and faculty members alike were, "Why don't faculty members take the time to take a metacognitive approach to problem solving?" and "Why don't faculty members talk about the entire problem-solving rather than expecting students merely to learn by example?" If instructors were to clarify for themselves the most efficient approaches for solving problems, this might enhance their teaching and student problem solving as well. As a result, emphasis in the preparation of physics teacher candidates should be placed on the metacognitive processes involved in problem solving. It also bodes well for a structured problem-solving process. A more systematic analysis of and approach to problem-solving difficulties in all areas of physics teaching promises to pay dividends.

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Appendix A

Think Aloud Problems

Please use a "think aloud" protocol as you solve the following problems. Use a separate sheet of paper for each problem. Clearly label each problem with the corresponding numbers below. A calculator is provided. Take the magnitude of the acceleration due to gravity (g) to be equal to 9.8 m/s^2. Below are formulas for your use.

$$\bar{v} = \frac{x}{t}$$

$$v = v_o + at$$

$$\bar{v} = \frac{(v_o + v)}{2}$$

$$x = v_o t + \frac{1}{2}at^2$$

$$v^2 - v_o^2 = 2ax$$

where x is the distance traveled by an object during a time t, with constant acceleration a, initial speed v_o, final speed v, and average speed $\bar{v}$.

1. A bullet is shot from a rifle with a speed of 160 m/s. If the barrel of the gun is 0.8 m in length, what is the average speed of the bullet while in the barrel assuming constant acceleration? For how long is the bullet in the barrel?
2. A “dragster” accelerates uniformly from rest to 100 m/s in 10 s. How far does it go during this interval?
3. A toy rocket is shot straight upward from ground level with an initial speed of 49 m/s. How long does it take the rocket to return to earth? Assume the absence of wind resistance.
4. A landing commercial airliner, upon “reversing” its engines, uniformly slows from 150 m/s to 30 m/s using 1,800 m of runway. What is the acceleration of the plane during this procedure?
5. A little girl glides down a long slide with a constant acceleration of 1 m/s^2. If the girl gives herself an initial speed of 0.5 m/s and the slide is 3 m long, what is her speed upon reaching the bottom of the slide? How long does it take her to reach the bottom of the slide?

Appendix B

Interview Questions

For novices and experts:

1. What is the first thing you search for in a problem statement?
2. What is the first thing you do after determining what you are to find?
3. Do you follow any particular pattern or procedures when you solve physics problems? If so, please explain.

For novices only:

4. When you have difficulties solving a physics homework problem, what do you do?
5. What use do you make of examples when attempting to solve problems with which you are having problems?
6. How did you learn to solve physics problems?

For experts only:

7. How do you teach your introductory physics students how to solve physics problems?
8. Do you ever talk about the problem-solving process? If so, what do you say?

Appendix C

Coding Plan for Observations of Physics Problem Solving

1. Visualize the problem.
    - draw a sketch
    - identify the known variables and constraints
    - restate the question
    - identify the general approach to the problem
2. Describe the problem in physics terms.
    - use identified principles to construct idealized diagram
    - symbolically specify the relevant known variables
    - symbolically specify the target variable
3. Plan a solution.
    - start with the identified physics concepts and principles in equation form
    - apply the principles systematically to each type of object or interaction
    - add equations of restraint that specify any special conditions
    - work backward from the target variable until you have determined that there is enough information to solve the problems
    - specify the mathematical steps to solve the problem
4. Execute the plan.
    - use the rules of algebra to obtain an expression for the desired unknown variable
    - instantiate the equation with specific values to obtain a solution
    - solve the equation for the desired unknown
5. Check and evaluate.
    - check - is the solution complete?
    - check - is the sign of the solution correct?
    - check - does the solution have the correct units?
    - check - is the magnitude of the answer reasonable?
6. Makes an Error.
    - makes error in solution of algebraic equation
    - makes error in statement of fact
7. Expresses Confusion.
    - admits confusion
    - expresses doubt
    - expresses anger
    - admits inability / gives up