Moment of Inertia Inquiry Lab

Draft Guidelines for TA

By Carl Wenning

Required background: Students must be able to solve problems involving that Atwood machine. PHY 110 students should be able to employ calculus for specific derivations; PHY 108 and PHY 105 students need not see the derivations. All students should be familiar with *DataStudio,* and know how to calibrate sensors.

A dry-run laboratory experiment was conducted with a student earlier; total time consumed was just under two hours. The experimental moment of inertia for the ring had less than 1% error compared with the theoretical value.

1) Give students a meter stick. Have them twist it back and forth with no masses attached and then with two equal masses attached at equal distances from the center. Ask, “Did you notice the change in resistance to rotational motion?” “In linear motion, there is a similar resistance to motion caused by mass. What do we call that form of resistance to motion?” (inertia) “What might we call this form of resistance to circular motion?” (rotational inertia, a.k.a. moment of inertia)

2) Ask, “How do we measure the inertia associated with linear motion?” (determine the mass, because mass an inertia are proportional)

3) Ask, “Does the moment of inertia depend only on mass?” Conduct some form of experiment(s) to see if there is some form of dependency other than that of mass.

4) Students experiment with meter stick using equal masses arranged at small then large distances from the pivot point. Ask, “Does it make any difference how the mass is distributed along the meter stick?” “Does the resistance to rotational motion change?” “In a qualitative sense, how so?”

5) Ask, “Is there a similar connection between speed, acceleration, and so on in linear motion as compared to circular motion? If there is, please describe.” Help students develop an analog between rotational and linear motions by having them complete the following chart. (torque and force, linear acceleration and angular acceleration, distance and angle, velocity and angular velocity)

Linear motion	**Circular motion**
force, F	torque, □
speed, v	angular speed, □
acceleration, a	angular acceleration, □
mass, m	moment of inertia, I
distance, s	angle, □
F = ma	□ = I□
s = r□ (standard radian formula)	

v = rω (a time derivative of above equation)
a = rα (a time derivative of above equation)
Fr = τ

6) Have students hypothesize as to which other factors, if any, might influence the moment of inertia and then conduct experiments to see if there is any sort of relationship.

7) Have students begin to create an experiment in which they determine the form of the relationship between moment of inertia, I, and the mass, m. Students first distinguish between independent, dependent, and control variables. Give a bit of release time and then Socratically question students in such a way that they are led to work through the following analysis where *T (= F)* is the tension on the string and other terms are defined above:

$$\tau = I\alpha$$

$$Tr = I\alpha$$

$$I = \frac{Tr}{\alpha}$$

$$\sum F = ma$$

$$mg - T = ma$$

$$T = m(g - a)$$

$$I = \frac{mr(g - a)}{\alpha}$$

$$\alpha = \frac{a}{r}$$

$$I = mr^2\left(\frac{g}{a} - 1\right)$$

8) Students then are released to find the relationship between *I* and *m*:

$$I \quad m$$

9) The students' graph will be of the form y = mx + b. Ask, "What does the y-intercept represent?" (the moment of inertia of the rotational unit). Have them subtract this value of b from each determination and re-plot the graph. This will show that the moment of inertia is directly proportional to mass (for a given mass configuration).

10) Help students to realize from the above that moments of inertia are <u>additive</u> in nature.

11) Have students create an experiment in which they determine the form of the relationship between the moment of inertia, *I,* and the distance, *r,* of a given set of counter-balancing masses. Before this form is achieved, students must first subtract the moment of inertia of the rotational unit and then re-plot.

$$I \propto r^2$$

12) Help students realize that the only reasonable form of relationship between *I, m,* and *r* is of the form:

$$I \propto mr^2$$

13) Help students realize that the complete form of the relationship for a “non-point-source distribution of matter” is as follows:

$$I = \sum_{1}^{n} m_i r_i^2$$

14) If appropriate, use calculus to derive the theoretical form of *I* for a ring; otherwise, give students formula for a ring. Have students calculate the moment of inertia for the ring using the derived formula (note: the + sign is not an error):

$$I_{disk} = \frac{1}{2} M(R_1^2 + R_2^2)$$

15) Have students determine *I* for combination of ring and rotational unit, and then subtract *I* for rotational unit to find *I* of the ring. **Warnings:** (1) Make certain that the sensor is properly calibrated before using a string on the pulley! (2) To obtain a greater accuracy for the acceleration, be derive acceleration from the slope of the linear speed versus time graph. This gives acceleration to the fourth decimal place instead of only to the second, and is critical for high accuracy.

16) Have students determine % error. A run by the “professor” resulted in less than 1% error.

17) Have students account for error to the best of their ability.

18) Have students complete lab report following guidelines including grading criteria.