Meany Means

Most of us tend to give very little thought to the word "mean" as in "average". Actually, there are several different types of means and each has its own use. We are probably most familiar with the *arithmetic* (pronounced *air-ith-mét-tic*) mean that is nothing more than the sum of all terms divided by the number of terms. The average or "mean value" of the terms 5, 6, and 10 is 21/3 = 7. Arithmetic means are suitable when all components contribute equally to the mean and can be weighted identically. That is not always the case however, especially in some areas of physics.

The arithmetic mean does not provide the correct information in situations that require weighting, for instance in averages involving <u>constant</u> rates. For example, if a truck goes on a round trip following the same route there and back totaling 100 miles with a constant speed of 50 miles per hour on the outbound leg and 25 miles per hour on the return leg, the average speed is NOT 37.5 miles per hour [(50*mph*+25*mph*)/2]. This is so because the TIMES of travel will be different for each leg of the trip. On the outbound portion of the trip, the time required to go 50 miles is only one hour. On the return portion of the trip, the travel time required to go 50 miles is two hours due to traveling at half the former speed. The "true average speed" is one in which the total travel time is the same as if the truck had traveled the whole distance at that average speed. A simple arithmetic mean of the two speeds does not give the true average speed because it does not take into account the times spent at those individual speeds. The 25*mph* portion of the trip should be double weighted due to the two hours in comparison to the 50*mph* portion of the trip at only one hour.

When weighted on the basis of time spent, then one can determine the true average speed as follows:

$$\text{rate}_{\text{average}}*(\text{total time}) = \text{rate}_1*\text{time}_1 + \text{rate}_2*\text{time}_2$$
$$v_{\text{averge}}*3hr = 25mph*2hr + 50mph*1hr$$
$$v_{\text{average}} = (50mi + 50mi)/3hr$$
$$v_{\text{average}} = 100mi/3hr$$
$$v_{\text{average}} = 33\,{}^1/_3 mph$$

There is another way of looking at this. The true average speed, $\overline{v}$, is defined as the total distance divided by the total time. That is,

$$\overline{v} = \frac{d_{total}}{t_{total}}$$

In the example above the total distance is 100*mi.* The total time is 3*hr.* Thus, the true average speed will be 100*mi*/3*hr* or $33\,{}^1/_3$*mph.* This is significantly different from the *arithmetic* mean of 37.5*mph.*

Believe it or not, there is yet another way to get the true average that utilizes weighted means. It's called the technique of the *harmonic* mean. In situations involving rates, the harmonic mean is another way to determine the correct, time-weighted average rate. The harmonic mean is defined as follows:

$$H = \frac{n}{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\right)}$$

where r_n represents the rates of the various components, and n the number of component. In a round trip, rate equals the speed and $n = 2$. For instance, if a vehicle travels a certain distance at a constant speed of

6*m/s* and then the same distance again at a constant speed of 4*m/s,* then its average speed is the *harmonic* mean that has a value of 4.8*m/s*. The calculation is as follows:

$$H = \frac{n}{\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\right)} = \frac{2}{\left(\frac{1}{6m/s} + \frac{1}{4m/s}\right)} = 4.8m/s$$

This same approach applies to more than two segments given a series of sub-trips at different constant speeds if each sub-trip covers the same *distance.* Then the true average speed is the *harmonic* mean of all the sub-trip speeds.

Now, let's look at another example. An airplane flies three legs of an equilateral triangle (all legs the same length). If the plane flies a constant ground speed of 150*mph* on the first leg, a constant ground speed of 200*mph* on the second leg, and constant ground speed of 250*mph* on the third leg, what is the average ground speed over the course of the flight? (Note that you are not given any distances or times.)

$$H = \frac{3}{\left(\frac{1}{150mph} + \frac{1}{200mph} + \frac{1}{250mph}\right)} = 191.5mph$$

This is significantly different from an arithmetic mean of 200*mph.* The process also helps you solve a problem that otherwise is not soluble without a knowledge of individual distances and times for each leg of the trip.

If and only if each sub-trip takes the same amount of *time*, then the average speed will be the *arithmetic* mean of all the sub-trip speeds. This is rarely the case in physics.