Working with Units

Scientists, if they are careful, always use units when making calculations. Using units can be critically important in avoiding errors. For instance, in 1999 the Mars Climate Orbiter crashed into the surface of Mars as a result of a metric-imperial units mix up. A navigation error arose because the contractors for craft's thrusters did not use metric (SI) units to express their performance. As a result, a multi-million-dollar mission was ruined and NASA publicly embarrassed. This problem would have been avoided if units had been checked.

Now, a student's mistakes are not going to cause such a problem, but if units are used when making calculations, they often can serve as a check on the calculation. For instance, if someone calculates a speed in meters per sec (m/s) and the units turn out to be seconds per meter (s/m), clearly there is something wrong with the calculation. For the purposes of this course, always use units throughout your calculations. Do not merely slap expected units on a calculation made without units.

Note the procedures for working with units in the example below. Speed is calculated by dividing distance traveled by time. That is, $s = d/t$. If $d = 45m$ (meters) and $t = 15s$ (seconds), then the calculation is done as follows:

$$s = \frac{d}{t} = \frac{45m}{15s} = \frac{3.0m}{s} \text{ or } 3.0m/s$$

If one were to mistakenly define speed as time divided by distance, then the answer for the speed would turn out to be one second per three meters or one-third second per meter, clearly not the units expected for a speed. Obviously, the units tell the person conducting the calculation that an error has been made.

Consider a more complex use of units involving squared terms and complex fractions of the form

$$\frac{a}{b} \bigg/ \frac{c}{d} = \frac{ad}{bc}$$

A bicyclist is moving at an original speed of $v_o = 2m/s$ and decelerates (negative acceleration) at a uniform rate of $a = -0.5m/s^2$. How far does the bicyclist go (d) in going from $v_o = 2m/s$ to a complete stop ($v = 0m/s$)? The governing relationship for this problem is the following equation that is then solved for d.

$$v^2 - v_o^2 = 2ad$$

$$d = \frac{v^2 - v_o^2}{2a} = \frac{-v_o^2}{2a}$$

$$d = \frac{-4m^2}{s^2} \bigg/ 2\frac{-0.5m}{s^2} = \frac{-4m^2s^2}{-1ms^2} = 4m$$

Note how units are simplified: $m^2/m = m$ and $s^2/s^2 = 1$ for instance. Also note how negative values in the numerator are canceled by negative values in the denominator (following proper algebraic rules). Always reduce units to their simplest form.