

Conversion Factors

The use of conversion factors will not be confusing so long as certain rules are adhered to rigidly. These rules can best be shown with the use of examples:

Problem: Convert 3 (assumed an integer) inches (*in*) to millimeters (*mm*).

The conversion factor is $1\text{in} = 25.4\text{mm}$ precisely. This can be rewritten in two ways:

$$\frac{1\text{in}}{25.4\text{mm}} = 1 \quad \text{or} \quad \frac{25.4\text{mm}}{1\text{in}} = 1$$

Now, we can multiply any quantity by 1 without changing its *essential* value. We can multiply 3 inches by 1 and still have 3 inches. If we choose to rewrite 1 in one of the above forms, we can convert a distance in one system of measure (British) to that of another (metric). The numerical value will not be the same because the unit of measurement will change. However, the actual distance will be the same in either system of measurement. The question is, which one of the above two ways should we choose to write 1 so as to successfully convert from one system of measurement to another?

If we choose to write 1 in the first way, we have:

$$3\text{in} = 3\text{in} \times 1 = 3\text{in} \times \frac{1\text{in}}{25.4\text{mm}} = \frac{0.118\text{in}^2}{\text{mm}}$$

The result is in mixed units that have no direct physical meaning and is a combination of both metric and British systems. If we choose to write 1 in the second way, we have:

$$3\text{in} = 3\text{in} \times 1 = 3\text{in} \times \frac{25.4\text{mm}}{1\text{in}} = 76.2\text{mm}$$

Note that the inches unit has canceled; the result is purely metric.

What happens if one needs to convert, say, cubic centimeters (cm^3) into cubic meters (m^3)? Consider the following problem.

Problem: Convert 300cm^3 into cubic meters.

The conversion factor is $100\text{cm} = 1\text{meter}$. Now, one cannot merely multiply 300cm^3 by $1\text{m}/100\text{cm}$ to arrive at the answer. If this were done, the answer would come out in $\text{cm}^2 \times \text{m}$ and, again, mixed units. To handle this sort of complication, we must rewrite 1 as 1^3 . One cubed is the same as one. For example:

$$\begin{aligned} 300\text{cm}^3 &= 300\text{cm}^3 \times 1^3 = 300\text{cm}^3 \times \left(\frac{1\text{m}}{100\text{cm}}\right)^3 \\ &= 300\text{cm}^3 \times \frac{1\text{m}^3}{1 \times 10^6\text{cm}^3} = \frac{300\text{m}^3}{10^6} = 3 \times 10^{-4}\text{m}^3 \end{aligned}$$

When using conversion factors, always pay meticulous attention to the units and their associated powers. In doing so, you need never make an error in converting from one system to another.