Dimensional Analysis

Scientists sometimes will use a process called dimensional analysis to predict the form of the relationship between variables in a system. While this does NOT constitute a theory base, it does give scientists a better understanding of what to expect experimentally, and helps them to design scientific experiments. Carefully observe the following process of dimensional analysis for a cart accelerating down a fixed inclined plane.

Let's say that a scientist observes the motion of the cart on the fixed incline, and after conducting a few qualitative experiments concludes that the distance of the cart is a function of its acceleration and the time. Assume that the cart accelerates from rest when the stopwatch starts. What is the expected form of the relationship between distance, acceleration, and time?

The form of this relationship can be predicted using an approach known as dimensional analysis. This process is based on the knowledge that the distance (*d*) is a function of the acceleration (*a*) and the time (*t*). That is,

$$d = f(a,t)$$

Now, if *d* (expressed in meters, *m*) is related to both *a* (expressed in meters per second squared, m/s^2) and *t* (expressed in seconds, *s*) in some form, then the only thing missing is a proportionality constant and the powers of the variable terms. Write a proportionality applying power terms *x* and *y* to *a* and *t* respectively Replace variables by units and solve for the power terms as, in this case, two simultaneous equations with two unknowns. Working backward, then find the form of the equation and insert proportionality constant. (Note that if the unit of time, *s*, is present on the right side of the equation, it must also be present on the left side of the equation. Place s^0 on the left side of the equation in step two below as shown; s^0 is equal to 1 and done not affect the equality.)

$$d \propto a^x t^y$$

replacing variables with units

$$m^1 s^0 \propto \left(\frac{m}{s^2}\right)^x s^y$$

simplifying

$$m^1 s^0 \propto m^x s^{y-2x}$$

equating exponents on *m* and *s*

$$1 = x$$

$$0 = y - 2x$$

and solving simultaneous equations

$$y = 2x$$

hence, $y = 2$

thus, $d \propto a^1 t^2$

or, $d = kat^2$

Experiments can be conducted or theoretical work performed to find the value of the constant, *k*. In reality, *k* equals 1/2 giving the familiar kinematic equation

$$d = \frac{1}{2}at^2$$