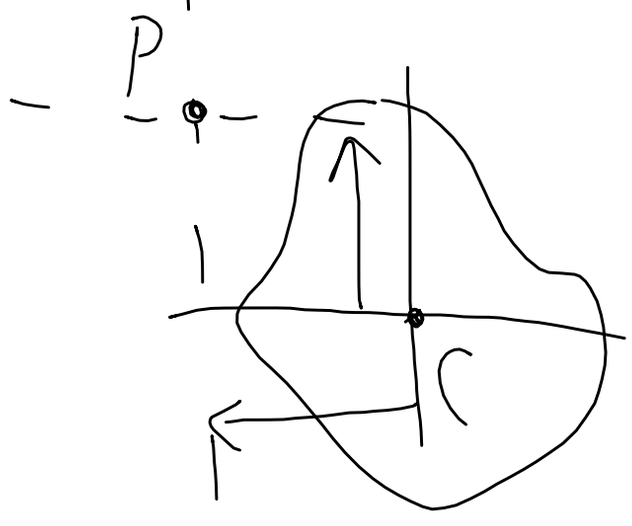
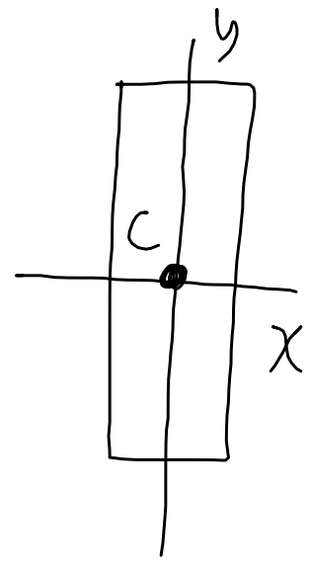
PAT for $\underline{\underline{I}}_{xy}$

$$\underline{\underline{I}}_{xy,P} = \underline{\underline{I}}_{xy,C} + A\,\bar{x}\,\bar{y}$$

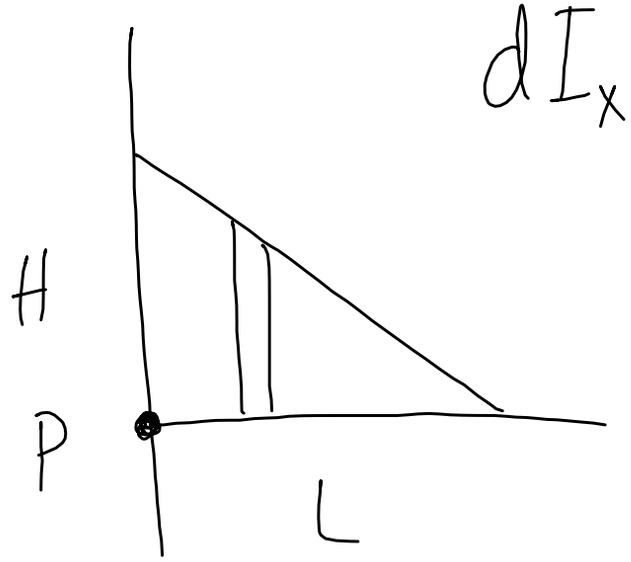
$\bar{x}$ $\bar{y}$ locates P from C



$$I_{xy,P} = I_{xy,C} + A\,\bar{x}\,\bar{y}$$



$$dI_{xy,P} = \underbrace{dI_{xy,C}}_{=0} + dA\,\bar{x}\,\bar{y}$$



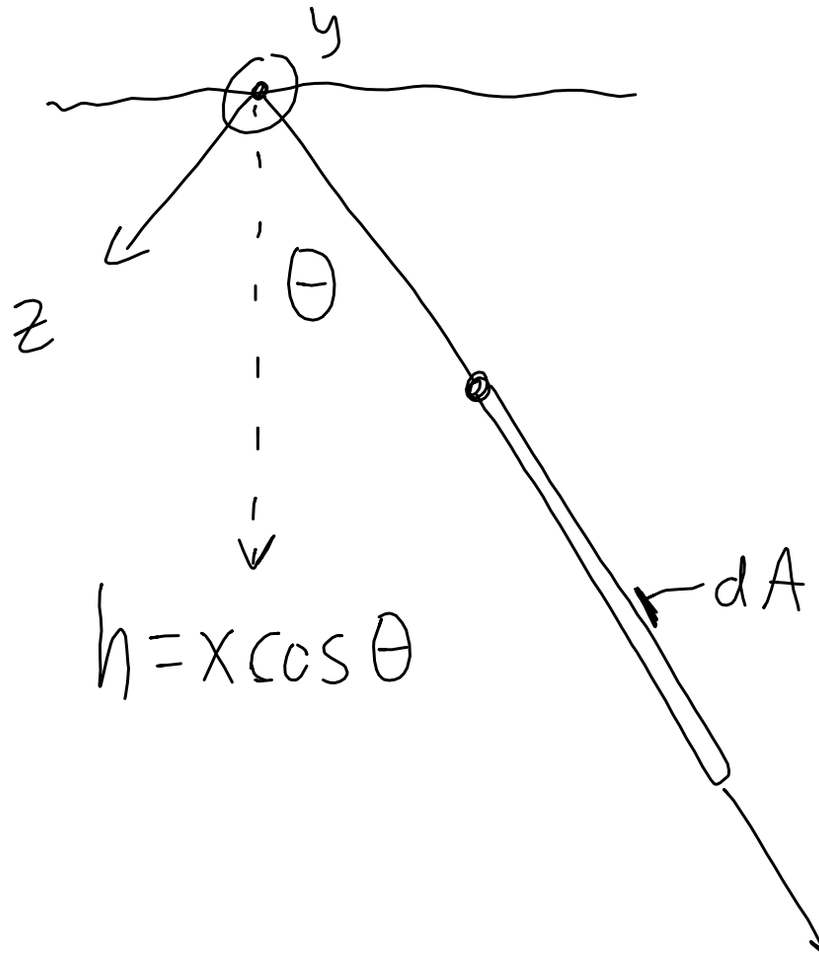
$dA = \left(H - \frac{H}{L}x\right)dx$

$\bar{x} = x$

$\bar{y} = \dfrac{\left(H - \frac{H}{L}x\right)}{2}$

$$I_{xy,P} = \int_0^L x\,\frac{\left(H - \frac{H}{L}x\right)}{2}\left(H - \frac{H}{L}x\right)dx = \frac{1}{2}\int_0^L x\left(H^2 - \frac{2H^2}{L}x + \frac{H^2}{L^2}x^2\right)dx$$

$$= \frac{1}{2}\left[\frac{H^2L^2}{2} - \frac{2}{3}H^2L^2 + \frac{H^2L^2}{4}\right] = \frac{H^2L^2}{24}$$



$$dF = p\,dA = \rho g h\,dA$$

$$F_R = \int \rho g x \cos\theta\,dA$$

$$= \rho g \cos\theta \int x\,dA$$

$$= \rho g \cos\theta\ x_c A$$

where to put it?
next page...

$$= \rho g h_c A$$

$$= p_c A$$

Find location $\vec{r}_{OP}$ so that

$$\vec{r}_{OP} \times \vec{F}_R = \int \vec{r} \times d\vec{F}$$

$$(x_p \hat{i} + y_p \hat{j}) \times (p_c A \, \hat{k}) = \int (x\hat{i} + y\hat{j}) \times p \, dA \, \hat{k}$$

$\hat{i}$: $y_p p_c A = \int y \, p \, dA$

$\hat{j}$: $-x_p p_c A = -\int x \, p \, dA$ ← start here

$$x_p = \frac{\int x\, p\, dA}{P_c A} = \frac{\int x\, \cancel{\rho g}\, x \cancel{\cos\theta}\, dA}{\cancel{\rho g}\, x_c \cancel{\cos\theta}\, A}$$

$$= \frac{\int x^2 dA}{x_c A} = \frac{I_{yo}}{x_c A} = \frac{I_{yc} + A x_c^2}{x_c A}$$

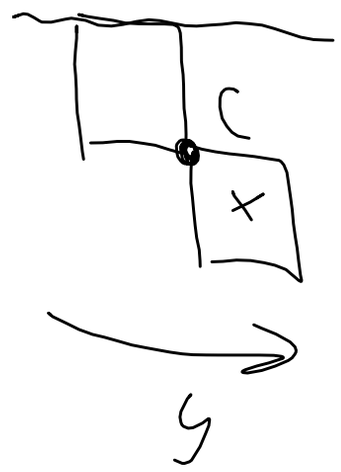
$$x_p = x_c + \frac{I_{yc}}{x_c A}$$

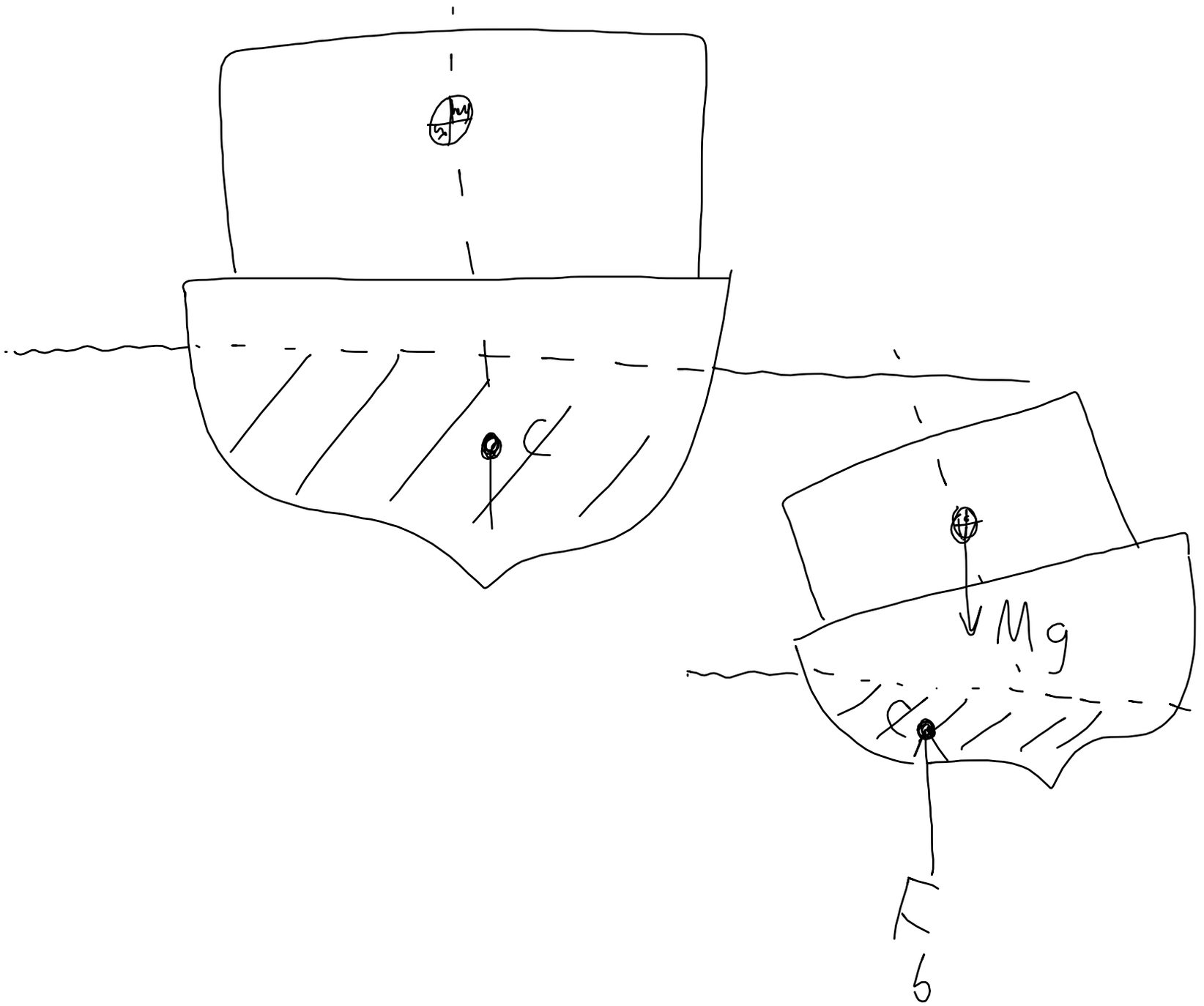
$$y_p P_c A = \int y P \, dA$$

$$y_p = \frac{\int y \cancel{\rho g} x \cancel{\cos\theta} \, dA}{\cancel{\rho g} x_c \cancel{\cos\theta} \, A} = \frac{\int x y \, dA}{x_c A}$$

$$= \frac{I_{xy,O}}{x_c A} = \frac{I_{xy,C} + A x_c y_c}{x_c A}$$

$$= y_c + \frac{I_{xy,C}}{x_c A}$$




C
Mg
T