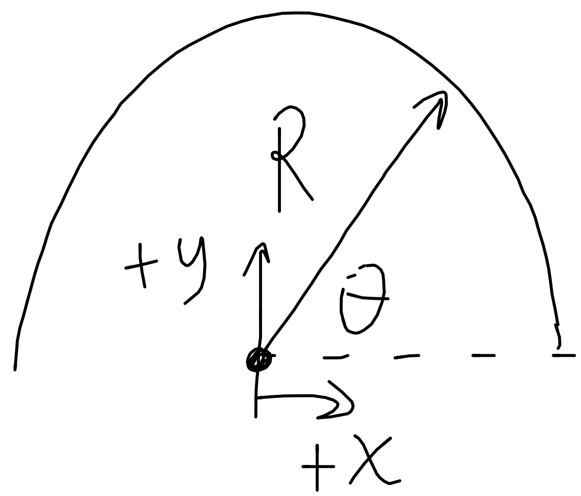
If Q is the "stuff" that's distributed

then

$$\int dQ \Rightarrow \text{total amount of stuff}$$

$$x_c = \frac{\int x \, dQ}{\int dQ}$$

$$Q \Rightarrow \ell, A, V, m$$



$x_c = 0$ by inspection

$\int x \, ds \qquad ds = R\,d\theta$

$x = R\cos\theta$

$$\int_0^{\pi} R\cos\theta \, R \, d\theta = R^2 \int_0^{\pi} \cos\theta \, d\theta$$

$$= R^2 \sin\theta \Big|_0^{\pi} = 0 \checkmark$$

$$y_c = ? \quad \frac{\int y\, ds}{\int ds}$$

$$\int ds = \int_0^\pi R\, d\theta$$

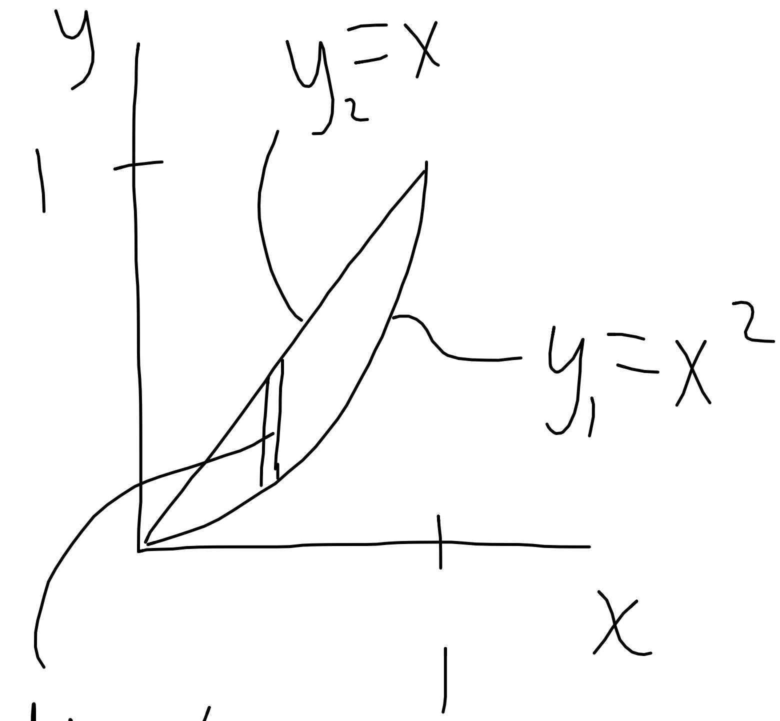
$$= R\pi$$

$$\int y\, ds = \int_0^\pi R \sin\theta \, R\, d\theta$$

$$= R^2 \left(-\cos\theta\right)\Big|_0^\pi = R^2\, 2$$

$$y_c = \frac{R^2\, 2}{R\pi}$$

$$= \frac{2R}{\pi}$$



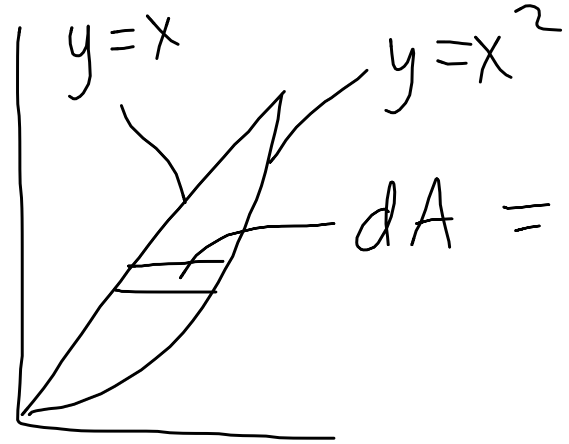
Where is centroid of bounded area?

$$x = \frac{\int x\,dA}{\int dA}$$

$$dA = (y_2 - y_1)\,dx$$

$$\int dA = \int_0^1 (x - x^2)\,dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{6}$$

$$\int x\,dA = \int_0^1 (x^2 - x^3)\,dx = \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{12} \qquad x_c = \frac{1}{2}$$



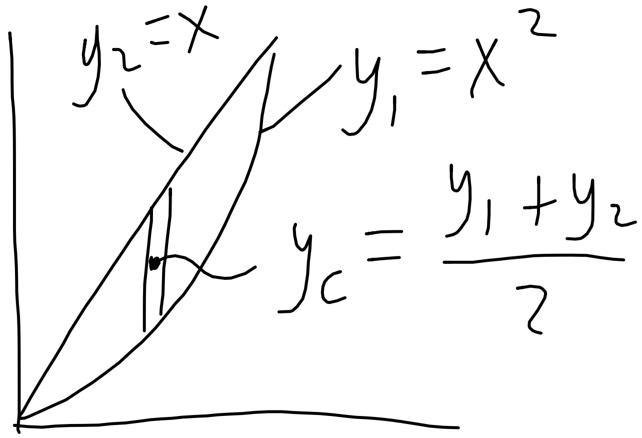
$$dA = (x_2 - x_1)\,dy = \left(y^{1/2} - y\right) dy$$

$$\int y\,dA = \int_0^1 y\left(y^{1/2} - y\right) dy = \int_0^1 \left(y^{3/2} - y^2\right) dy$$

$$= \frac{2}{5} y^{5/2} - \frac{y^3}{3} \Bigg|_0^1 = \frac{2}{5} - \frac{1}{3} = \frac{1}{15}$$

$$y_c = \frac{1/15}{1/6} = \frac{6}{15}$$

or

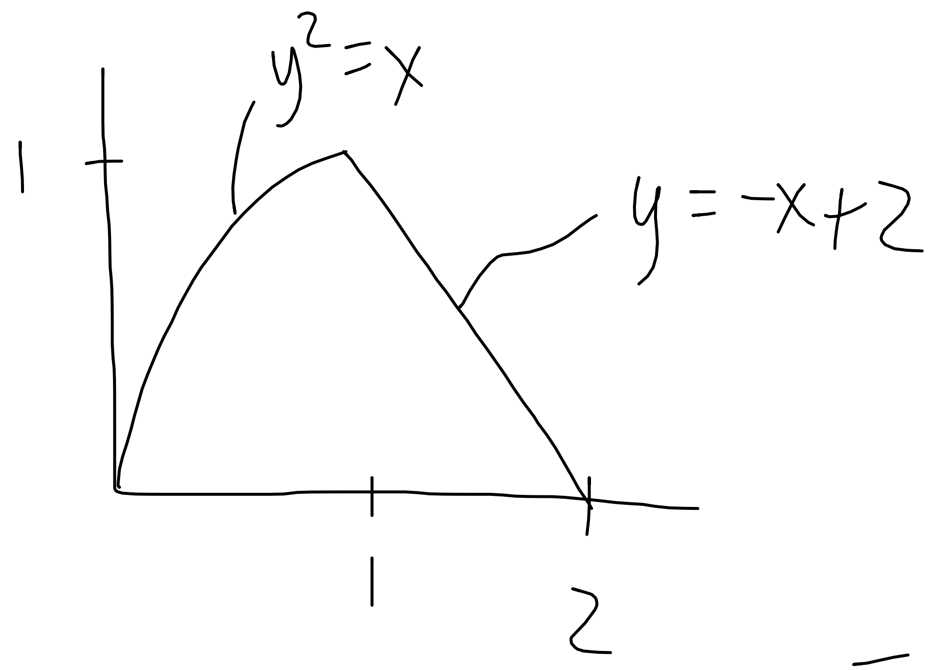


$$dA = (y_2 - y_1)\,dx$$

$$\int y\,dA = \int_0^1 \frac{y_1 + y_2}{2}(y_2 - y_1)\,dx$$

$$= \int_0^1 \frac{(y_2^2 - y_1^2)}{2}\,dx = \int_0^1 \frac{x^2 - x^4}{2}\,dx = \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$= \frac{1}{15} \checkmark$$



$$\int dA \Rightarrow \int dA_p + \int dA_L$$

$$= \int_0^1 x^{1/2}\,dx + \int_1^2 (-x+2)\,dx$$

$$= \frac{2}{3}x^{3/2}\Big|_0^1 + \frac{-x^2}{2} + 2x\Big|_1^2$$

$$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

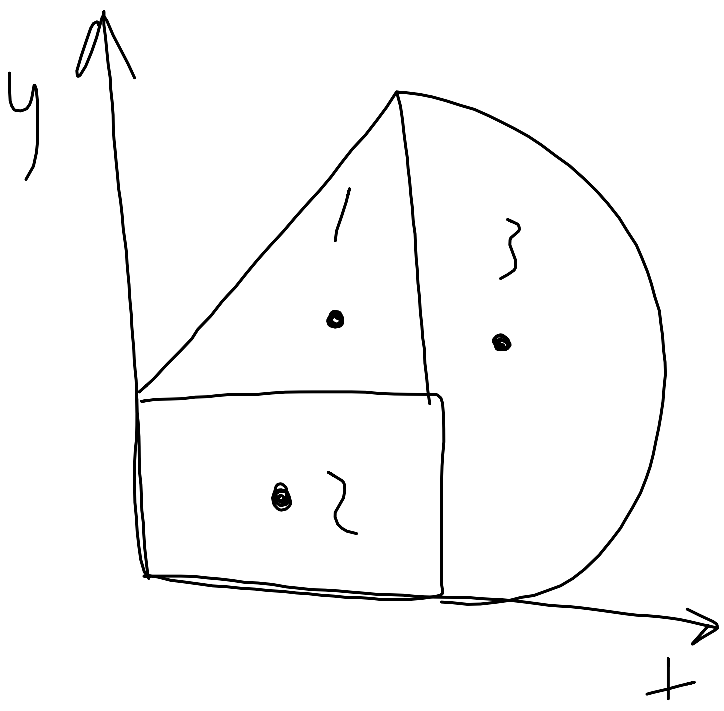
$$\int x\,dA$$

$$= \int_0^1 x^{3/2}\,dx + \int_1^2 (-x^2+2x)\,dx = \frac{2}{5}x^{5/2}\Big|_0^1 + \frac{-x^3}{3} + x^2\Big|_1^2$$

$$= \frac{2}{5} + \left[-\frac{8}{3} + 4 + \frac{1}{3} - 1\right] = \frac{16}{15}$$

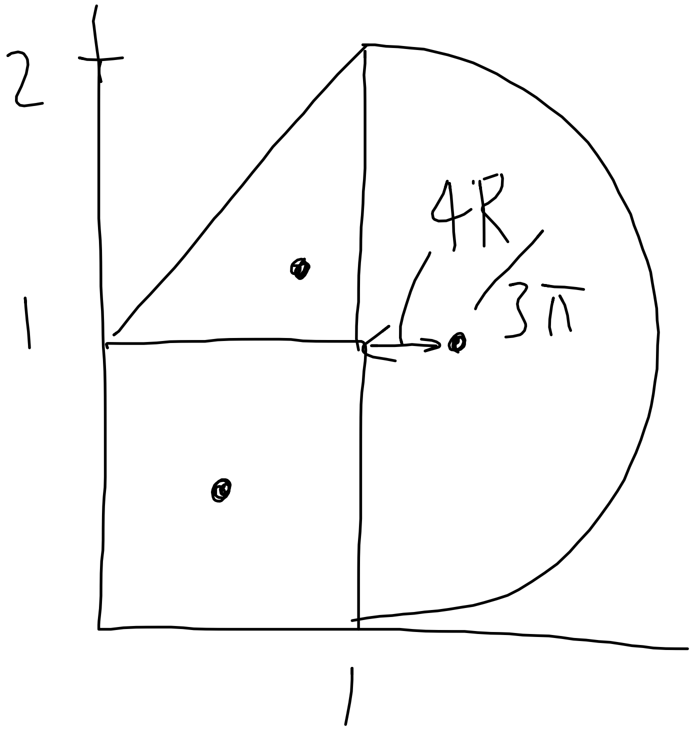
$$x_c = \frac{16/15}{7/6} = \frac{16(6)}{15(7)} = \frac{32}{35}$$

Rule of Composites



$$x_c = \frac{\sum x_{c_i} A_i}{\sum A_i}$$

Same fun
y_c



$$x_c = \frac{\sum x_i A_i}{\sum A_i}$$

$$= \frac{\frac{1}{2}(1) + \frac{2}{3}\left(\frac{1}{2}\right) + \left(1 + \frac{4(1)}{3\pi}\right)\frac{\pi(1)^2}{2}}{1 + \frac{1}{2} + \frac{\pi(1)^2}{2}}$$

$$x_c = 1$$