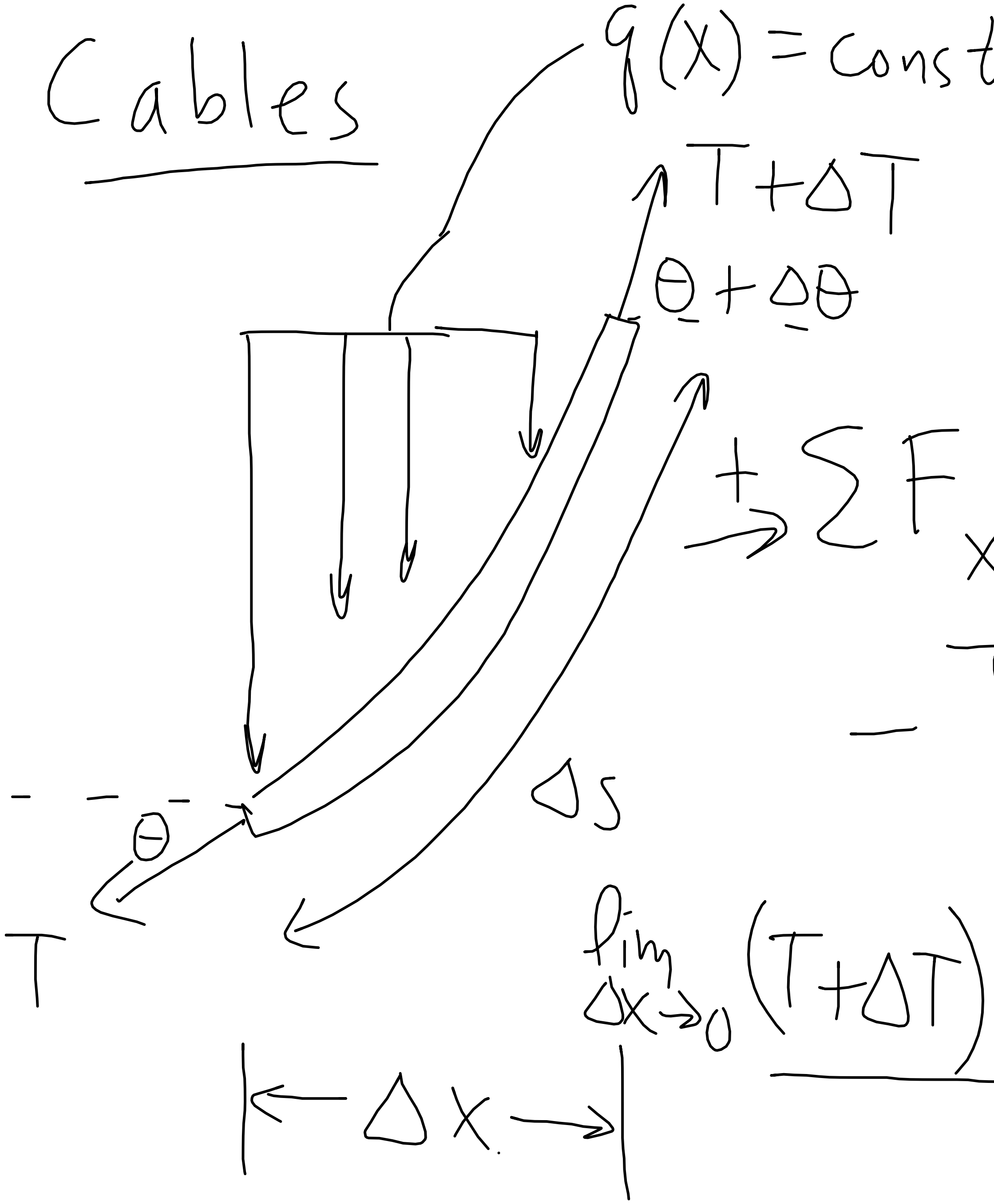
Cables



$q + \downarrow$

$$\xrightarrow{+} \sum F_x = (T+\Delta T)\cos(\theta+\Delta\theta) - T\cos\theta = 0$$

$$\lim_{\Delta x \to 0} \frac{(T+\Delta T)\cos(\theta+\Delta\theta) - T\cos\theta}{\Delta x}$$

$$= \frac{d}{dx}(T\cos\theta) = 0 \qquad \underbrace{T\cos\theta}_{T_H} = \text{const}$$

$$+\uparrow \sum F_y = (T+\Delta T)\sin(\theta+\Delta\theta) - T\sin\theta - q\,\Delta x = 0$$

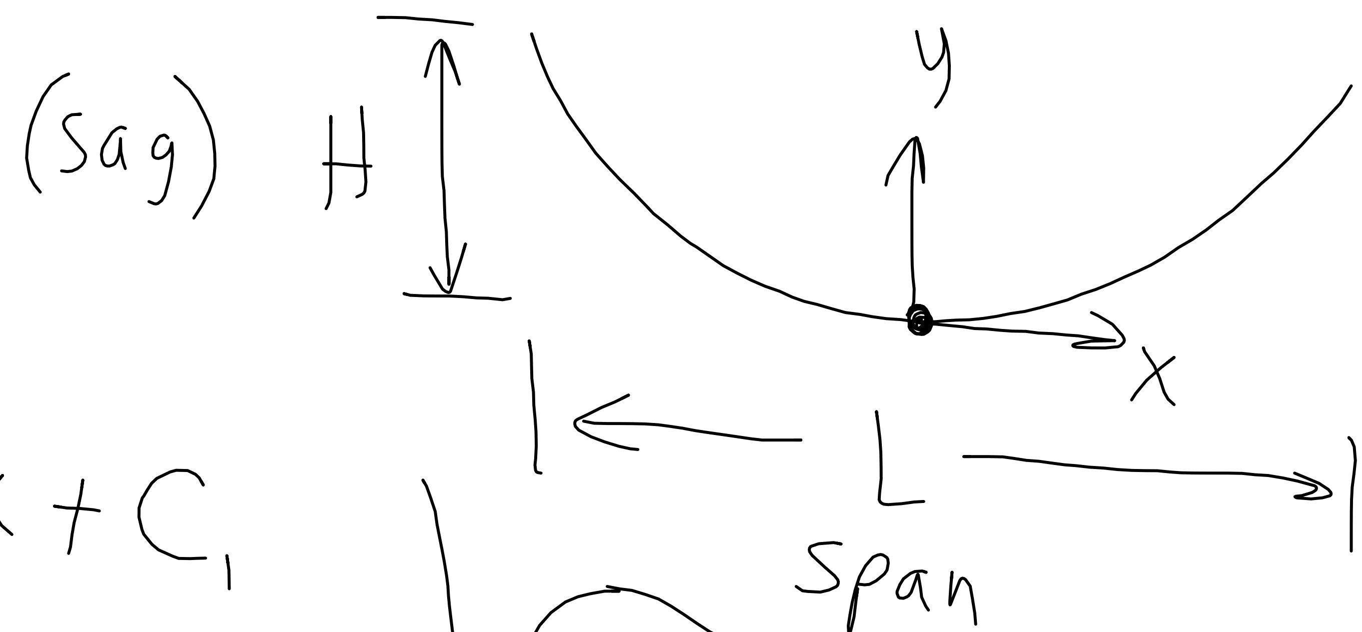
$$\lim_{\Delta x \to 0}\left[\frac{(T+\Delta T)\sin(\theta+\Delta\theta) - T\sin\theta}{\Delta x} - q\,\frac{\Delta x}{\Delta x}\right] = 0$$

$$\frac{d}{dx}\left(T\sin\theta\right) = q$$

$$\frac{dy}{dx}$$

$$\frac{d}{dx}\left[\frac{T_H}{\cos\theta}\sin\theta\right] = q \Longrightarrow \frac{d}{dx}\left(\tan\theta\right) = \frac{q}{T_H} \Rightarrow \frac{d^2y}{dx^2} = \frac{q}{T_H}$$

$$\frac{d^2y}{dx^2} = \frac{q}{T_H} \Longrightarrow \text{parabola}$$



$$\frac{dy}{dx} = \frac{q}{T_H}x + C_1$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 0 \quad \therefore C_1 = 0$$

$$y = \frac{q}{T_H}\frac{x^2}{2} + C_2 \Longrightarrow C_2 = 0$$

Recall $\frac{d}{dx}(T\sin\theta) = q$

$$T\sin\theta = qx + C_3$$

$\therefore$ But $T\sin\theta = 0$ @ $x = 0 \implies C_3 = 0$

$$T\sin\theta = qx$$

$$T = \sqrt{T_H^2 + (T\sin\theta)^2} = \sqrt{T_H^2 + q^2x^2}$$

$$T_{max} = \sqrt{T_H^2 + \frac{q^2L^2}{4}}$$

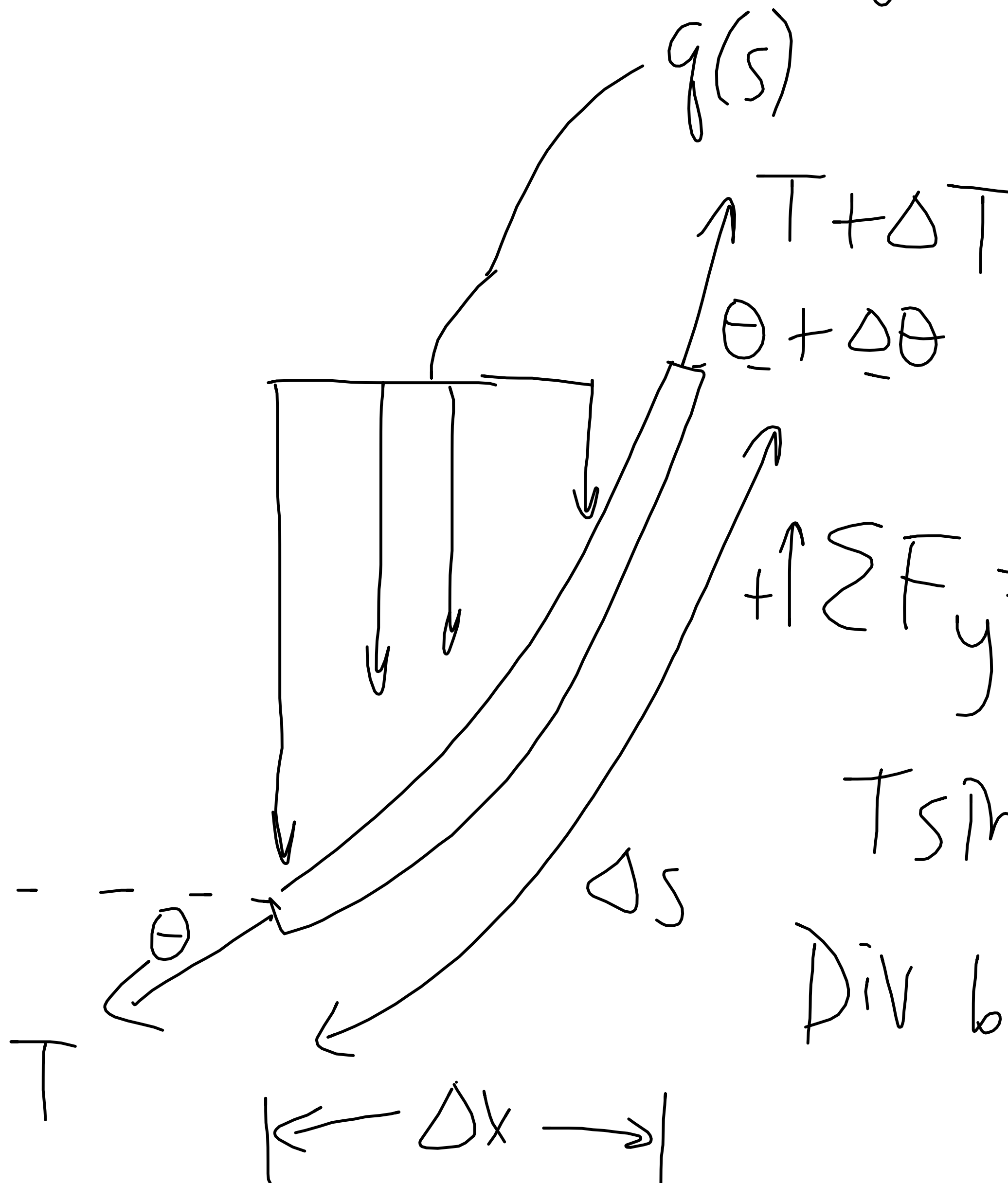
Since $y = \frac{g}{T_H}\frac{x^2}{2}$ $\quad H = \frac{g L^2}{T_H 8}$

$$T_H = \frac{g^2 L^2}{8H}$$

$$T_{max} = \sqrt{\frac{g^4 L^4}{64H^2} + \frac{g^2 L^2}{4}}$$

Now assume $q(s) \Rightarrow$ weight of cable

or ice on cable



$\sum F_x \Rightarrow T_H = \text{const}$

$+\uparrow \sum F_y = (T+\Delta T)\sin(\theta+\Delta\theta) -$

$T\sin\theta - q\Delta s = 0$

Div by Δx, $\lim\limits_{\Delta x \to 0}$

$$\lim_{\Delta x \to 0} \frac{(T+\Delta T)\sin(\theta+\Delta\theta) - T\sin\theta - q\Delta s}{\Delta x} = 0$$

$$\frac{d}{dx}(T\sin\theta) = q\frac{ds}{dx}$$

In infinitesimal limit

$$ds^2 = dx^2 + dy^2 \qquad ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$y' \equiv \frac{dy}{dx}$$

$$ds = \sqrt{1+y'^2}\, dx \quad \therefore \frac{ds}{dx} = \sqrt{1+y'^2}$$

$$T = \frac{T_H}{\cos\theta}$$

$$\frac{d}{dx}\left(T\sin\theta\right) = q\sqrt{1+y'^2} \qquad y' = \tan\theta$$

$$\frac{d}{dx}\left[T_H \tan\theta\right] = q\sqrt{1+y'^2}$$

$$T_H\, y'' = q\sqrt{1+y'^2}$$

$$y'' = \frac{q}{T_H}\cdot\sqrt{1+y'^2} \quad \text{let } p = y'$$

$$p' = \frac{g}{T_H}\sqrt{1+p^2}$$

$$\frac{p'}{\sqrt{1+p^2}} = \frac{g}{T_H}$$

$$\int \frac{dp}{\sqrt{1+p^2}} = \int \frac{g}{T_H}\,dx = \frac{g}{T_H}x + K_1$$

$$\ln\left(p + \sqrt{1+p^2}\right)$$

$$\ln\left(p+\sqrt{1+p^2}\right) = \frac{g}{T_H}x + K_1$$

$$p+\sqrt{1+p^2} = e^{\frac{g}{T_H}x+K_1}$$

$$1+p^2 = e^{\frac{2g}{T_H}x+2K_1} - 2pe^{\frac{g}{T_H}x+K_1} + p^2$$

$$p = \frac{e^{\frac{2g}{T_H}+2K_1} - 1}{2e^{\frac{g}{T_H}x+K_1}} = \frac{e^{\frac{g}{T_H}x+K_1} - e^{-\left[\frac{g}{T_H}x+K_1\right]}}{2} \Rightarrow \sinh$$