From last time,

$$p = \sinh\left(\frac{g}{T_H} x + K_1\right)$$

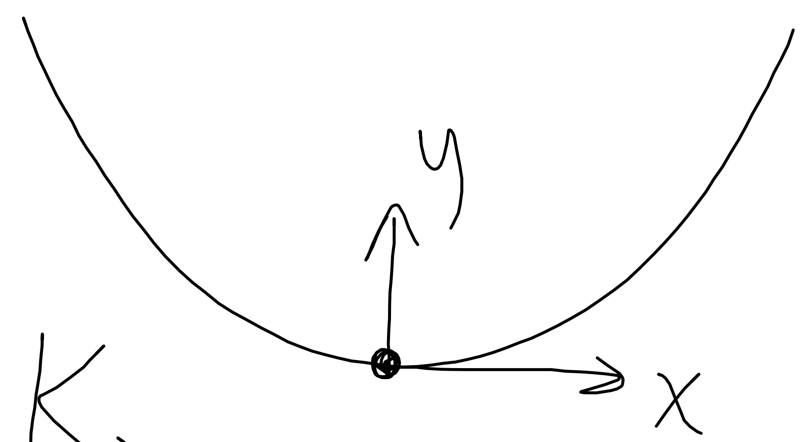
$$p = y' = \frac{dy}{dx}$$

$$y' = \sinh\left(\frac{g}{T_H} x + K_1\right)$$

$$y = \frac{T_H}{g}\left[\cosh\left(\frac{g}{T_H} x + K_1\right)\right]$$

$$y' = \sinh\left(\frac{q}{T_H}x + K_1\right)$$

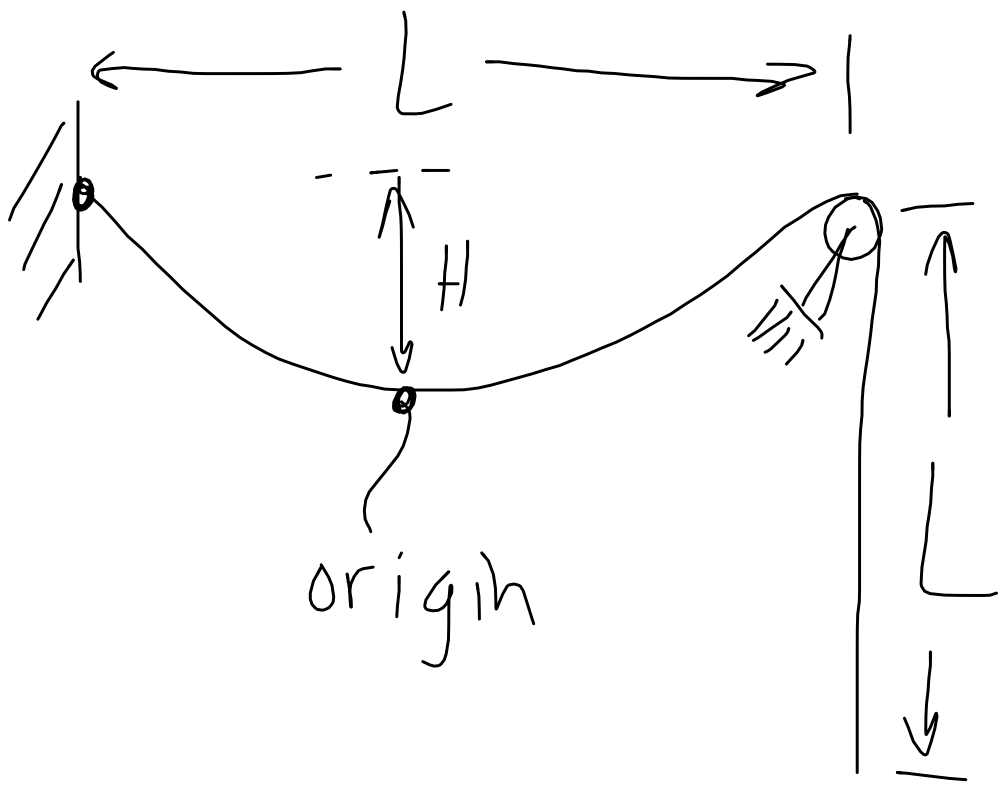
$$y = \frac{T_H}{q}\left[\cosh\left(\frac{q}{T_H}x + K_1\right)\right] + K_2$$



$$y' = 0 \ @\ x = 0 \implies K_1 = 0$$

$$y = 0 \ @\ x = 0 \implies K_2 = -\frac{T_H}{q}$$

$$y = \frac{T_H}{q}\left[\cosh\left(\frac{q}{T_H}x\right) - 1\right]$$



$g_0 =$ uniform wt./length

Find T_{max}, ℓ_{total} & sag (H)

$$T_{max} = T_{end} = g_0 L$$

$$y = \frac{T_H}{g_0}\left[\cosh\left(\frac{g_0}{T_H}x\right) - 1\right]$$

$$l_{total} = \int ds$$

of curved part

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

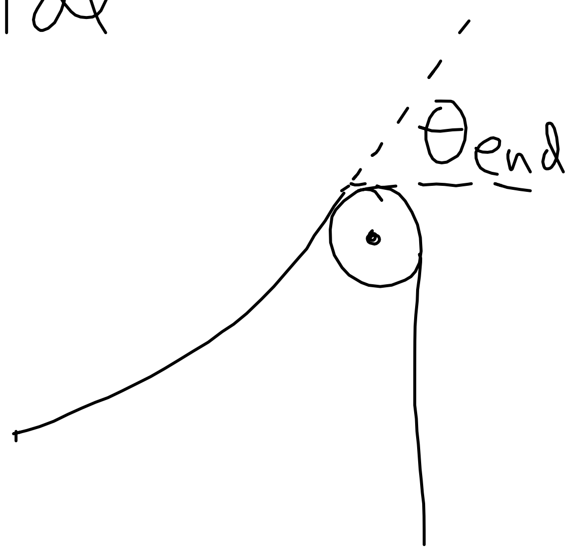
$$= dx\sqrt{1 + y'^2}$$

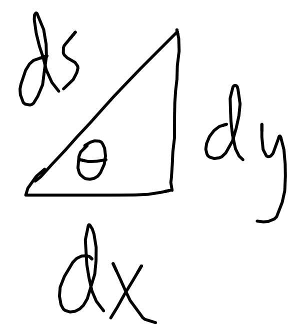
$$= \int_{-L/2}^{L/2} \sqrt{1 + \sinh^2\left(\frac{q_0}{T_H}x\right)}\, dx$$

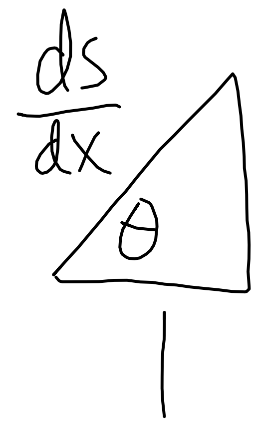
$$= \int_{-L/2}^{L/2} \cosh\left(\frac{q_0}{T_H}x\right) dx = 2\left[\frac{T_H}{q_0}\sinh\left(\frac{q_0}{T_H}x\right)\right]_0^{L/2}$$

$$\ell_{\text{total curved}} = 2\,\frac{T_H}{q_0}\,\sinh\left(\frac{q_0 L}{2T_H}\right)$$

Aside





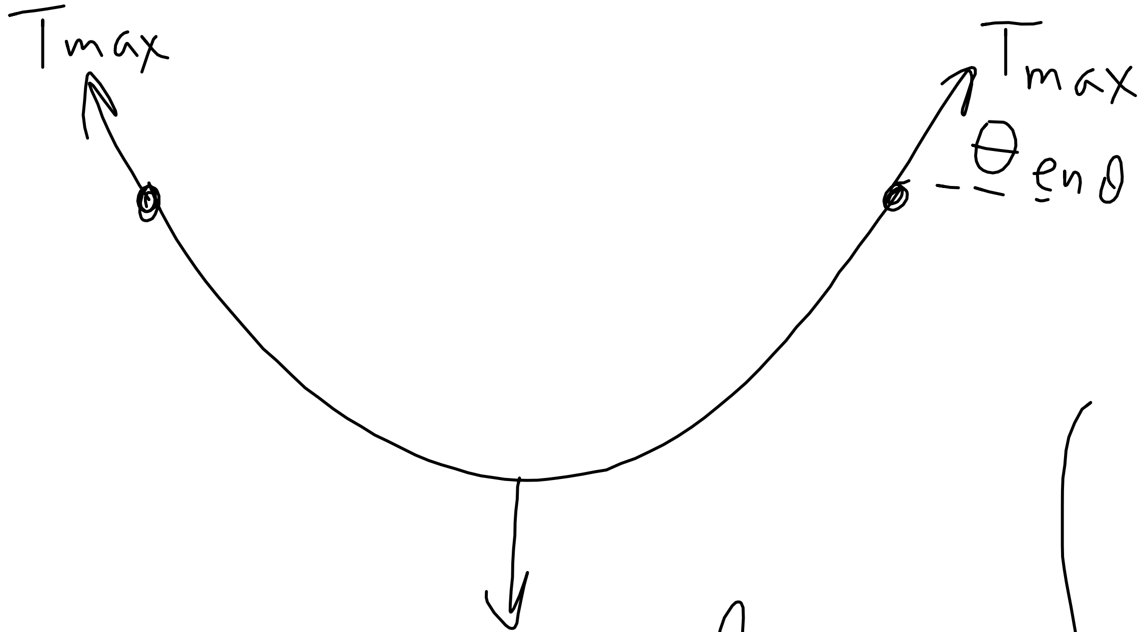


$$\frac{dy}{dx} = y'$$

$$\sin\theta_{end} = \frac{y'}{ds/dx} = \frac{y'}{\sqrt{1+y'^2}}$$

$$\sin\theta_{end} = \frac{\sinh\left(\frac{q_0}{T_H}x\right)}{\cosh\left(\frac{q_0}{T_H}x\right)} = \tanh\left(\frac{q_0}{T_H}x\right)\Bigg|_{x=\frac{L}{2}}$$

$$\sin\theta_{end} = \tanh\left(\frac{q_0}{T_H}\frac{L}{2}\right)$$



$$T_{max} = q_0 L$$

$$2 T_{max} \sin\theta_{end} = q_0 \, l_{curved}$$

$$W = q_0 \, l_{\substack{total\\curved}}$$

$$2 q_0 L \tanh\left(\frac{q_0}{T_H}\frac{L}{2}\right) = q_0 \, 2 \frac{T_H}{q_0} \sinh\left(\frac{q_0}{T_H}\frac{L}{2}\right)$$

Since $\tanh = \dfrac{\sinh}{\cosh}$,

$$2\,\frac{g_0 L}{2\,T_H} = \cosh\left(\frac{g_0}{T_H}\,\frac{L}{2}\right)$$

$$2\phi - \cosh(\phi) = 0$$

$$\phi \approx .589$$

$$\sin\theta_{end} = \tanh(.589) \qquad \theta_{end} = 31.95^\circ$$

$$T_H = T_{max}\cos\theta_{end}$$

$$= \frac{q_0 L}{1.179}$$

$$l_{curved} = 2\frac{T_H}{q_0}\sinh\left(\frac{q_0}{T_H}\frac{L}{2}\right) = 1.06L$$

For sag H

$$y \text{ @ } x = \frac{L}{2}$$

$$y = \frac{T_H}{q_0}\left[\cosh\left(\frac{q_0 x}{T_H}\right) - 1\right]$$

$$= .15L$$