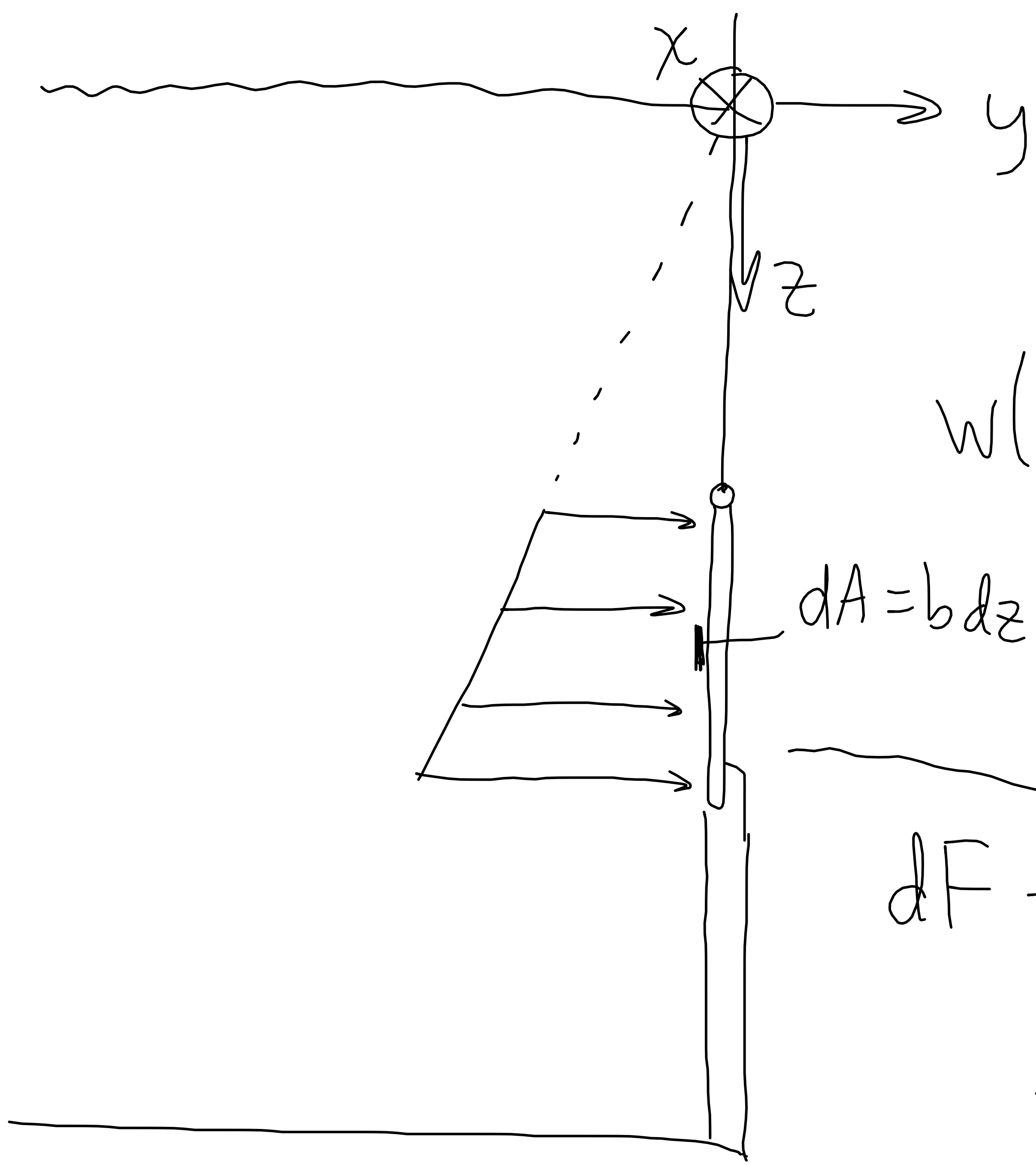


Assume gate has uniform width b

$$w(z) = p(z)\,b$$

$$= \rho g z b$$

called γ

$$dF = p(z)\,dA$$

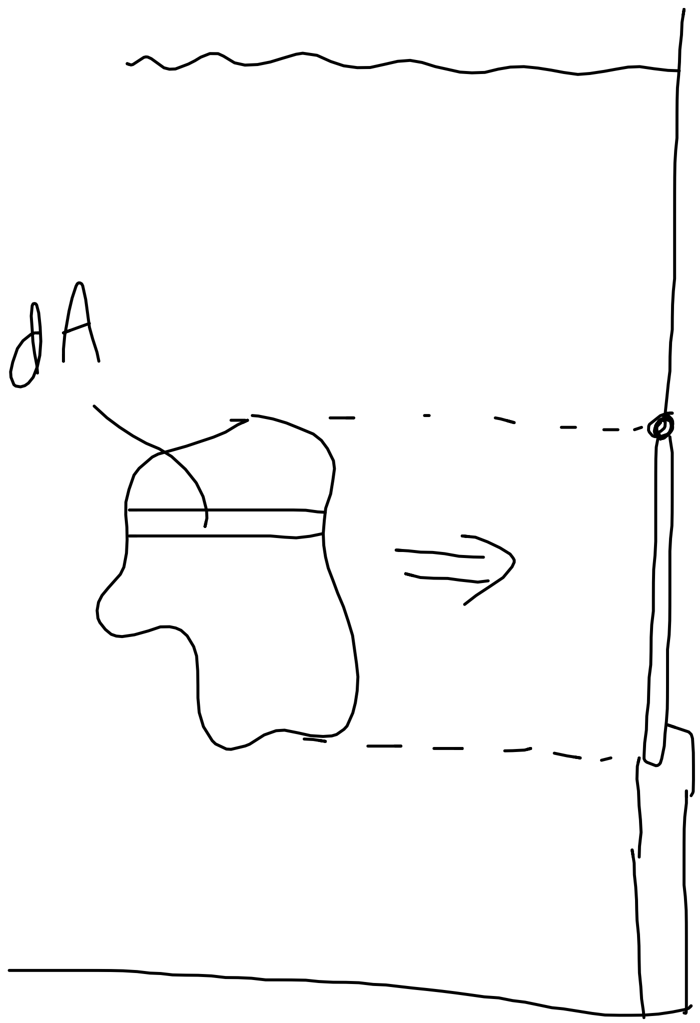
$$= \rho g z\,dA$$

$$F_R = \int \rho g z\,dA = \rho g \int z\,dA$$

$$F_R = \rho g \frac{\int z \, dA}{A_{gate}} A_{gate}$$

$$= \rho g z_c A_{gate}$$

$$= P_{\substack{\text{centroid} \\ \text{of} \\ \text{gate}}} A_{gate}$$



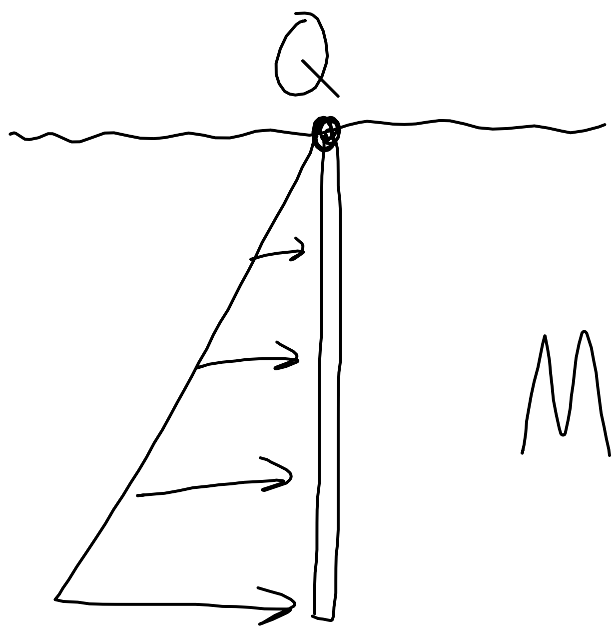
$$dF = \rho g z \, dA$$

$$F_R = p_c \, A_{gate}$$

Where?

Put it at centroid of volume under $p(x,z)$

Consider simpler problem



Uniform gate, width b

$$M_Q = \int z \, \rho g z \, dA$$

$$= \rho g \int z^2 \, dA$$

Area moment of inertia

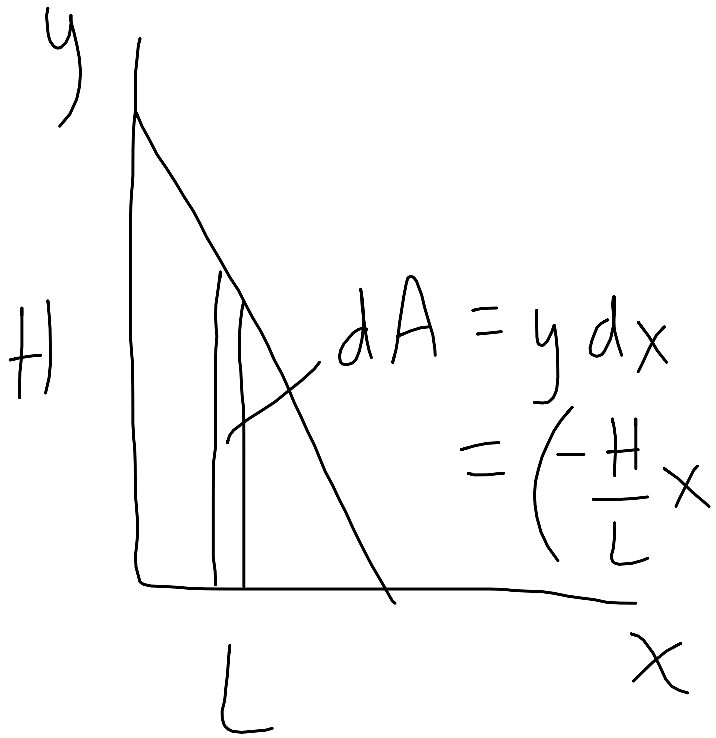
Moment of Inertia (for areas)

$$I_x = \int y^2 \, dA$$

y is dist $\perp$ from x-axis

$$I_y = \int x^2 \, dA$$

$$I_z = \int r^2 \, dA \qquad r^2 = x^2 + y^2$$



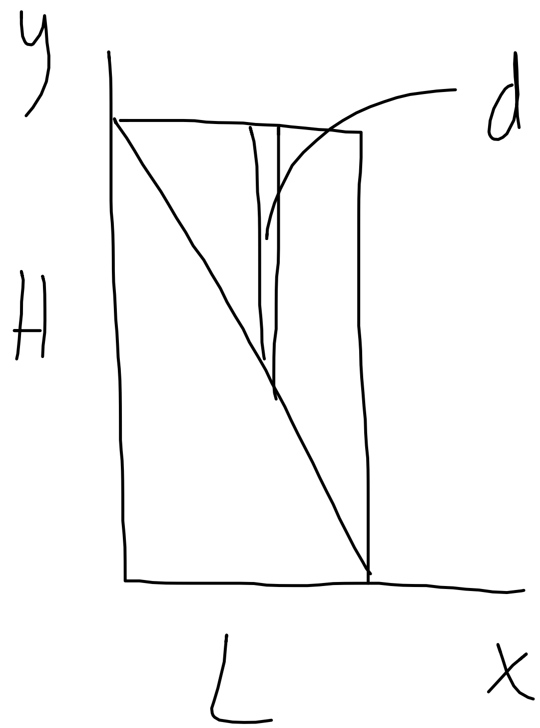
$dA = y\,dx$

$= \left(-\frac{H}{L}x + H\right)dx$

$$I_y = \int x^2\,dA$$

$$I_y = \int_0^L x^2\left[-\frac{H}{L}x + H\right]dx$$

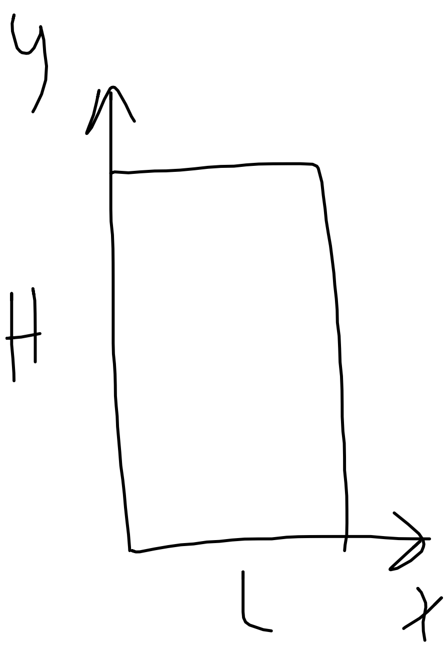
$$= -\frac{H}{L}\frac{x^4}{4} + H\frac{x^3}{3}\Bigg|_0^L = HL^3\left(-\frac{1}{4}+\frac{1}{3}\right) = \frac{HL^3}{12}$$



$$dA = \left\{ H - \left[-\frac{H}{L} x + H \right] \right\} dx$$

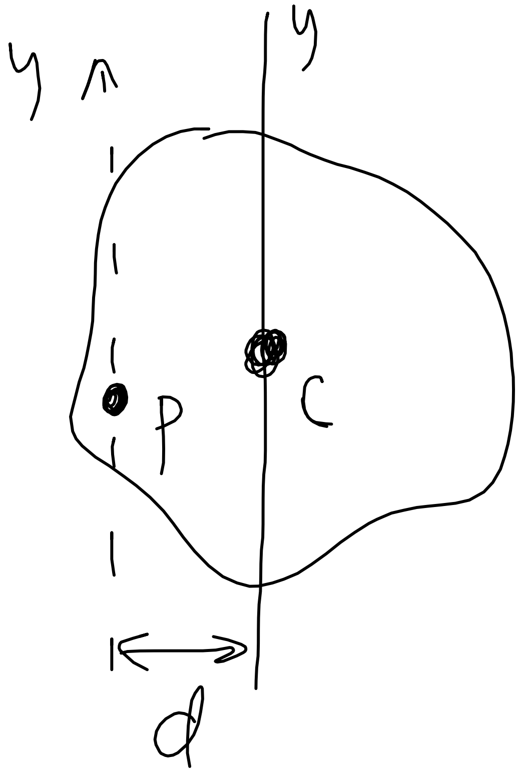
$$I_y = \int_0^L x^2 \left[H + \frac{H}{L} x - H \right] dx$$

$$= \frac{HL^3}{4}$$

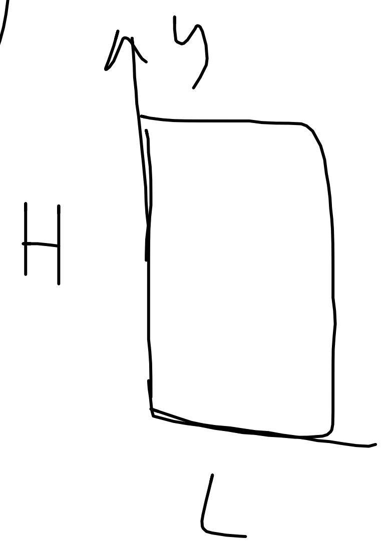


$$I_y = \frac{HL^3}{12} + \frac{HL^3}{4} = \frac{HL^3}{3}$$

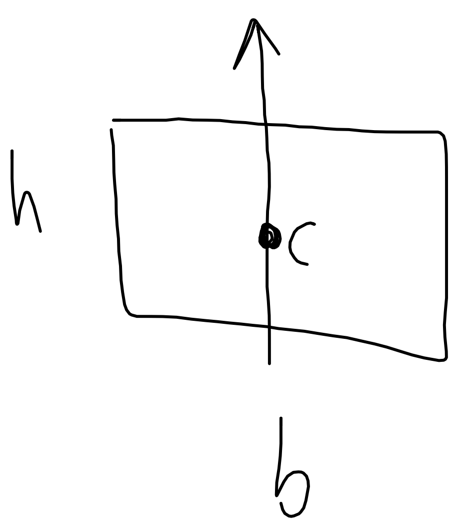
Parallel Axis Theorem



$$I_{y,p} = I_{y,c} + A d^2$$



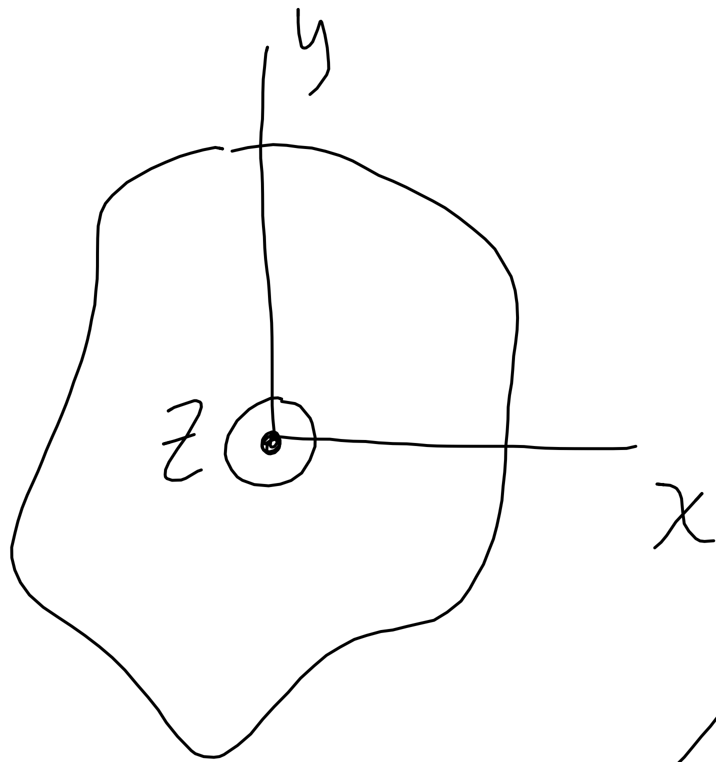
$$I_{y,0} = \frac{HL^3}{3} \rightarrow I_{yc} = \frac{HL^3}{12}$$



$$I_{y,c} = \frac{hb^3}{12}$$

check

$$\frac{HL^3}{3} - HL\left(\frac{L}{2}\right)^2 = \frac{HL^3}{12} \checkmark$$



Polar moment of inertia

$$I_z = \int r^2 \, dA$$

$$= \int (x^2 + y^2) \, dA$$

$$= \int x^2 \, dA + \int y^2 \, dA$$

$$= I_y + I_x$$

$$\vec{\tau} = I\vec{\alpha}$$ too simple

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xy} \\ \cdot & I_{yy} & \cdot \\ \cdot & \cdot & I_{zz} \end{pmatrix} \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix}$$