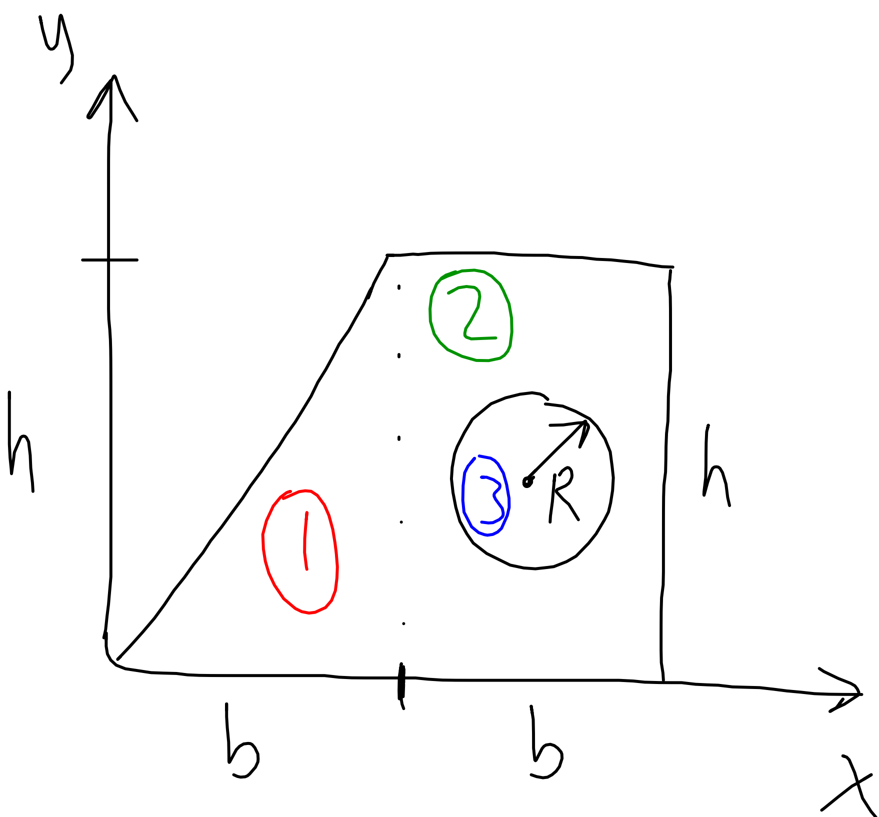
Moments of inertia $\Rightarrow$ composite rule

& Parallel axis



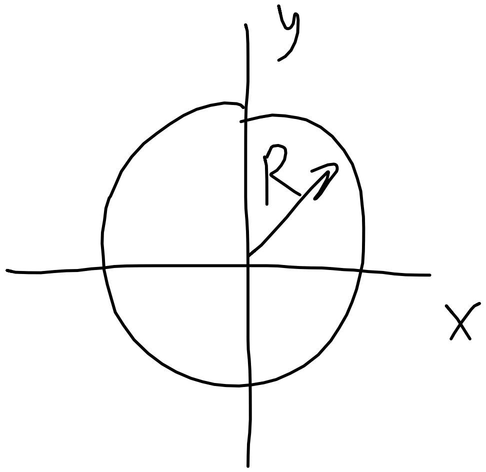
$$I_y = I_1 + I_2 - I_3$$

$$I_1 = I_{y,c} + A d^2$$

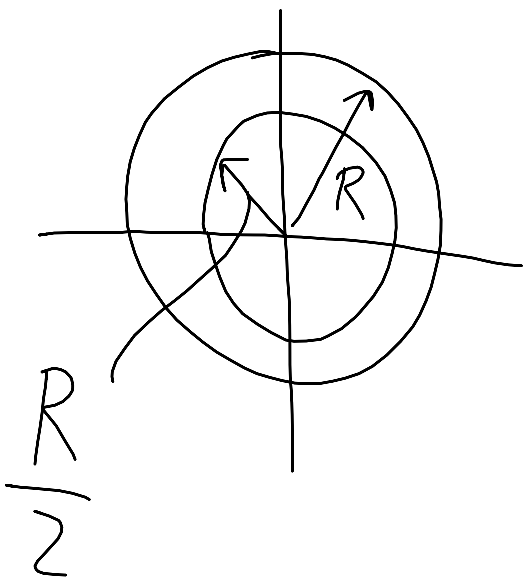
$$= \frac{hb^3}{36} + \frac{1}{2} bh \left(\frac{2b}{3}\right)^2$$

$$I_2 = \frac{hb^3}{12} + bh\left(\frac{3}{2}b\right)^2$$

$$I_3 = \frac{\pi R^4}{4} + \pi R^2 \left(\frac{3}{2} b\right)^2$$



$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$

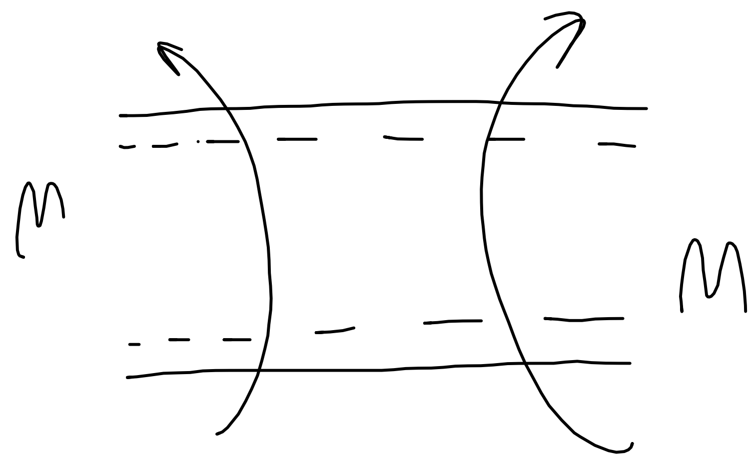


$$I_{xc} = \frac{\pi R^4}{4} - \frac{\pi \left(\frac{R}{2}\right)^4}{4}$$

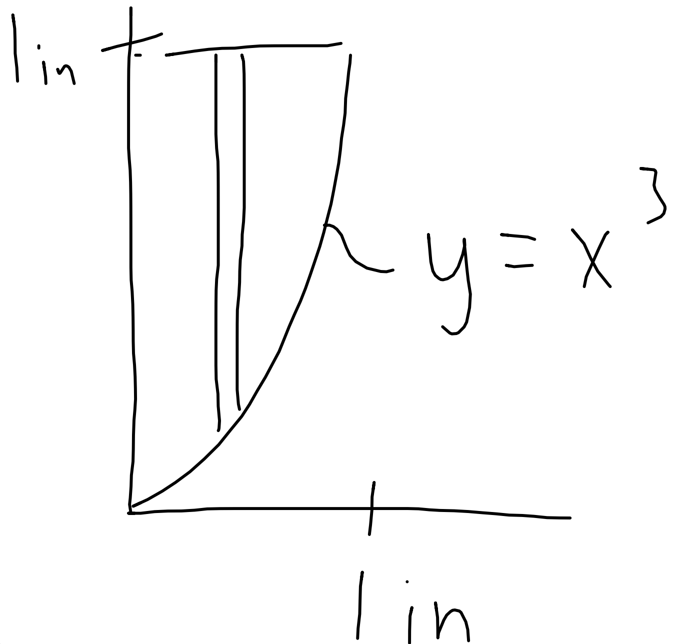
$$= \frac{\pi R^4}{4} + \frac{\pi R^4}{64} = \frac{15}{64} \pi R^4$$

$$= \frac{15}{16} \frac{\pi R^4}{4}$$

Recall



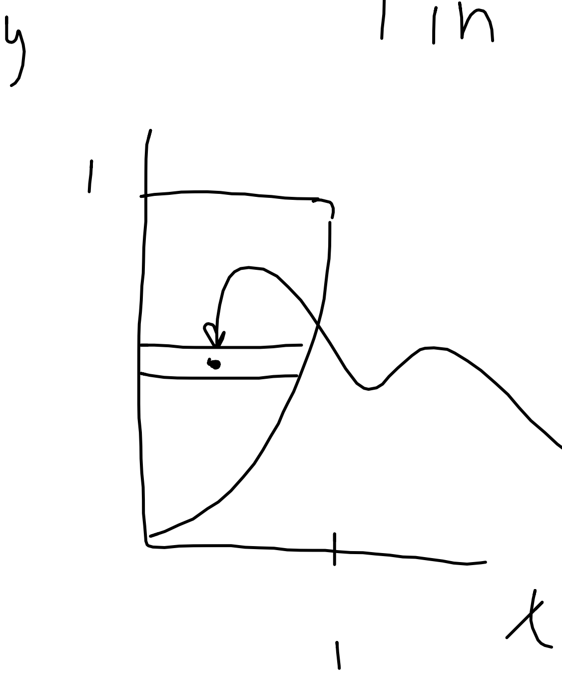
$$\sigma_{max} = \frac{M y_{max}}{I_x}$$



$$I_y = \int x^2 \, dA$$

$$= \int_0^1 x^2 (1 - x^3) \, dx$$

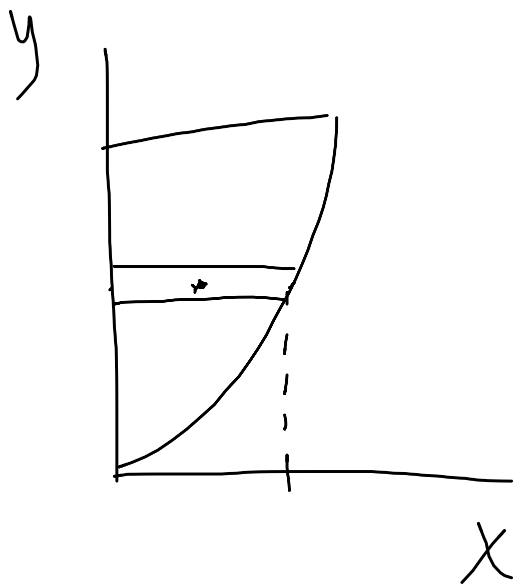
$$= \frac{x^3}{3} - \frac{x^6}{6} \Big|_0^1 = \frac{1}{6} \text{ in}^4$$



$$I_y = \int x^2 \, dA$$

?

But



For dA,

$$dI_y = dI_{y,c} + dA\left(\frac{x}{2}\right)^2$$

$$= \frac{x^3\,dx}{12} + x\,dy\left(\frac{x}{2}\right)^2 \;\ldots\ldots$$

Moral $\Rightarrow$ can't use same trick as for centroids!

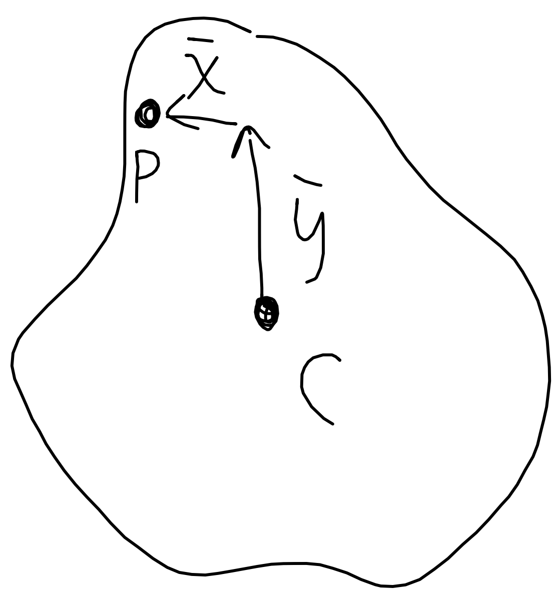
Recall

$$\vec{\tau} = I\vec{\alpha}$$

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ \cdot & I_{yy} & \cdot \\ \cdot & \cdot & I_{zz} \end{pmatrix} \begin{pmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{pmatrix}$$

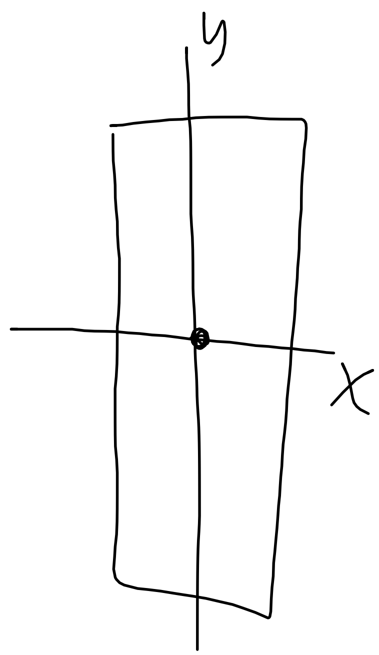
Product moment of inertia

$$I_{xy} = \int xy \, dA$$

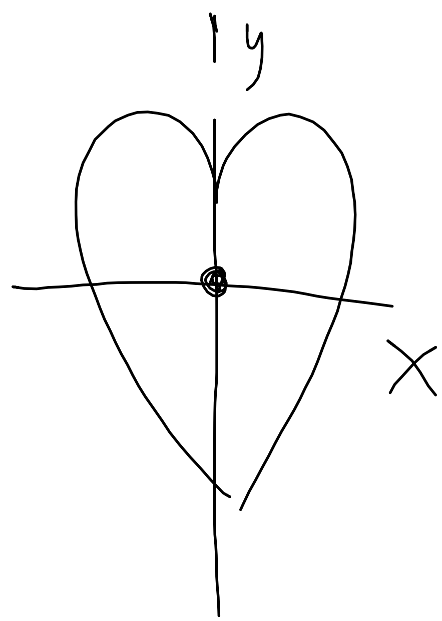


$$I_{xy,P} = I_{xy,C} + A\bar{x}\bar{y}$$

I_{xy} has symmetry rules!



$$I_{xy,c} = 0$$



$$I_{xy} = 0$$