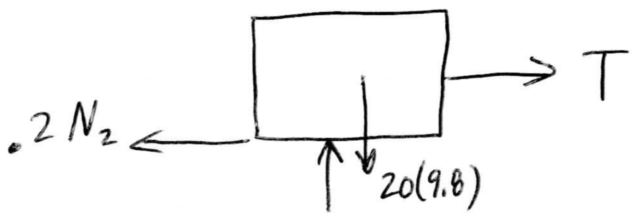
Exam 3 Sp 14

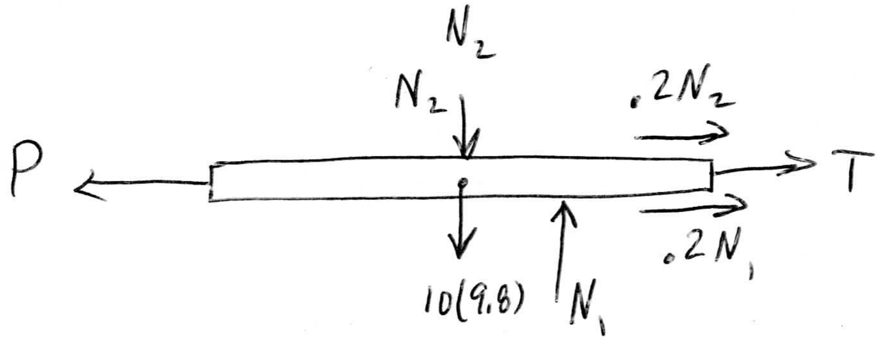
1.



By inspection

$N_2 = 20(9.8) = 196$

$T = .2(196) = 39.2$



For plate,

$+\uparrow \Sigma F_y = N_1 - 196 - 10(9.8) = 0 \qquad N_1 = 294$

$\overset{+}{\rightarrow} \Sigma F_x = 39.2 + .2(196) + .2(294) - P = 0$

$$\boxed{P = 137.2 \text{ N}}$$

2. Use vertical strip $\qquad dA = (1 + x^2 - x)\,dx$

$$A = \int dA = \int_0^1 (1 + x^2 - x)\,dx = x + \frac{x^3}{3} - \frac{x^2}{2}\Big|_0^1$$

$$= 1 + \frac{1}{3} - \frac{1}{2} = \frac{5}{6}\ \text{cm}^2$$

$$\int x\,dA = \int_0^1 (x + x^3 - x^2)\,dx = \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^3}{3}\Big|_0^1$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{1}{3} = \frac{5}{12}\ \text{cm}^3$$

$$x_c = \frac{\int x\,dA}{\int dA} = \frac{5/12}{5/6} = \boxed{\frac{1}{2}\ \text{cm}}$$

y_c for each vertical strip is at $\dfrac{1 + x^2 + x}{2}$

$$\int y\,dA = \int_0^1 \left(\frac{1 + x^2 + x}{2}\right)(1 + x^2 - x)\,dx$$

$$= \frac{1}{2}\int_0^1 \left(1 + \cancel{x^2} \cancel{- x} + x^2 + x^4 \cancel{- x^3} \cancel{+ x} \cancel{+ x^3} \cancel{- x^2}\right) dx$$

$$= \frac{1}{2}\int_0^1 (1 + x^2 + x^4)\,dx = \frac{1}{2}\left(x + \frac{x^3}{3} + \frac{x^5}{5}\Big|_0^1\right) = \frac{23}{30}$$

$$y_c = \frac{23/30}{5/6} = \boxed{\frac{23}{25}\ \text{cm}}$$

(For volume of revolution, $y_c = \frac{11}{10}$)

3. Left half rectangle

$$I_x = \frac{bh^3}{3} = \frac{3(10)^3}{3} = 1000 \text{ in}^4$$

Bottom right rectangle

$$I_x = \frac{bh^3}{3} = \frac{3(4)^3}{3} = 64 \text{ in}^4$$

Triangle

$$I_{xc} = \frac{bh^3}{36} = \frac{3(6)^3}{36} = 18 \text{ in}^4$$

$$I_x = I_{xc} + Ad^2 = 18 + \tfrac{1}{2}(3)(6)(6)^2 = 342 \text{ in}^4$$

Circle (hole)

$$I_{xc} = \frac{\pi R^4}{4} = \frac{\pi(2)^4}{4} = 4\pi \text{ in}^4$$

$$I_x = I_{xc} + Ad^2 = 4\pi + \pi(2)^2(4)^2 = 68\pi \text{ in}^4$$

$$I_x = 1000 + 64 + 342 - 68\pi$$

$$= 1192.4 \text{ in}^4$$

4. $I_{xy} = \int xy \, dA$

$$= \int_0^4 \int_0^{\sqrt{x}} x y \, dx \, dy$$

$$= \int_0^4 x \left[\frac{y^2}{2}\right]_0^{\sqrt{x}} dx$$

$$= \int_0^4 \frac{x^2}{2} dx$$

$$= \left.\frac{x^3}{6}\right|_0^4$$

$$= \frac{32}{3} \text{ in}^4$$

Or, use vertical strip + note that xy refers to location of centroid of dA

$$I_{xy} = \int_0^4 x y \, dA = \int_0^4 x \frac{\sqrt{x}}{2} \sqrt{x} \, dx = \int_0^4 \frac{x^2}{2} dx$$

$$= \left.\frac{x^3}{6}\right|_0^4 = \frac{64}{6} = \frac{32}{3} \text{ in}^4 \checkmark$$