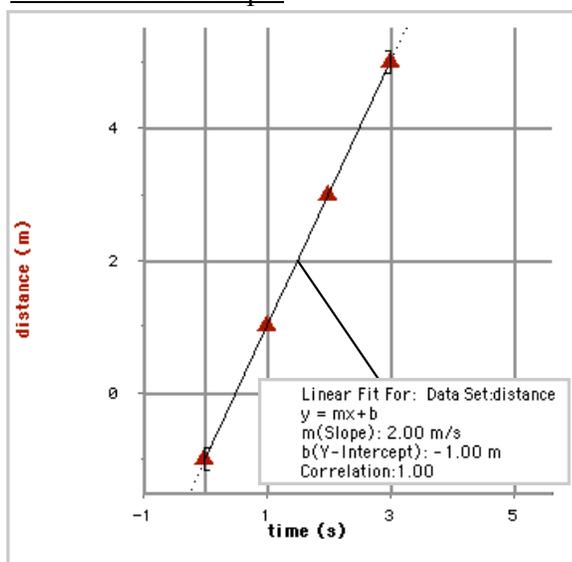


Deriving Relationships from Graphs

Graphing is a powerful tool for representing and interpreting numerical data. In addition, graphs and knowledge of algebra can be used to find the relationship between variables plotted on a scatter chart. Consider the following examples and procedures. Become familiar with these approaches, and attempt to thoroughly understand the processes illustrated here. You will be using these approaches regularly in introductory physics laboratory activities. Please note that this is merely an introduction to deriving relationships from graphs. There ultimately will be much more to learn.

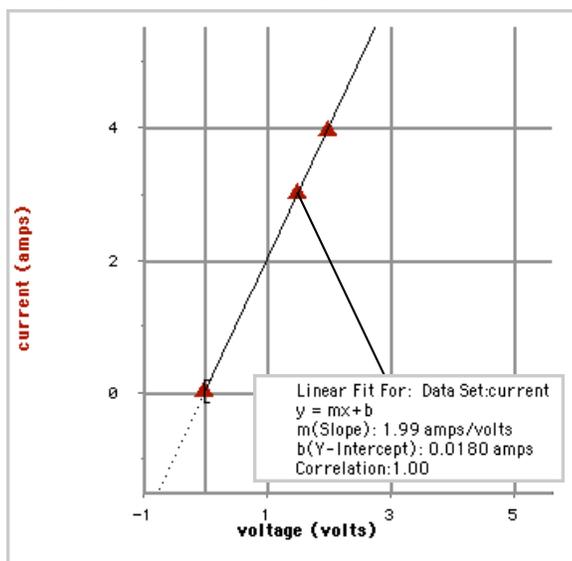
Linear Relationships:



The graph to the left depicts a linear relationship. That is, the data points all can be connected with a straight line. Note that the best-fit line does not pass through the origin; rather, it passes through the y -axis at $-1m$. How would one define the relationship between the values of the x and y coordinates in this relationship? It's done with the equation of a straight line, $y = mx + b$ where m is the slope and b is the value of the y -intercept – the value of y when $x = 0$. The slope, m , is found from $\Delta y/\Delta x = (y_2 - y_1)/(x_2 - x_1) = 2m/s$ in this graph. The value of b can be found from inspection as equal to $-1m$. The algebraic relationship is then represented as $y = mx + b$ where y is identified with the distance, x is identified with time, m the slope, and b the y -intercept. Thus, our relationship from the graph is:

$$\text{distance} = (2m/s)\text{time} - 1m.$$

Proportional Relationships:

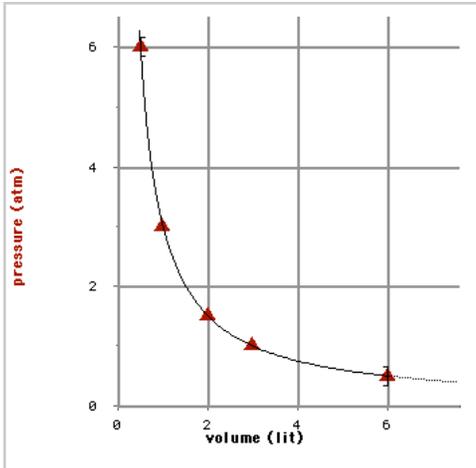


The graph to the left approximates the proportional relationship between voltage and current in a circuit. Note that a relationship is directly proportional only when the relationship is linear and passes through the origin. This is not quite the case in the current graph. The value of b is 0.0180 amps. The linear fit derived by the computer used to generate the graph performs an algebraic best fit to the data. The equation generated is the best mathematical fit, but the fit does not properly represent the physical reality of the situation. If no voltage is applied, then no current flows. If this is the case, the best-fit line must pass through the origin in what is known as a physical fit of the data. A physical fit demands $y = Ax$ as the model for the fit. Under the new analysis $m = 2\text{amps/volt}$. Then we find:

$$\text{current} = (2\text{amps/volt})\text{voltage}$$

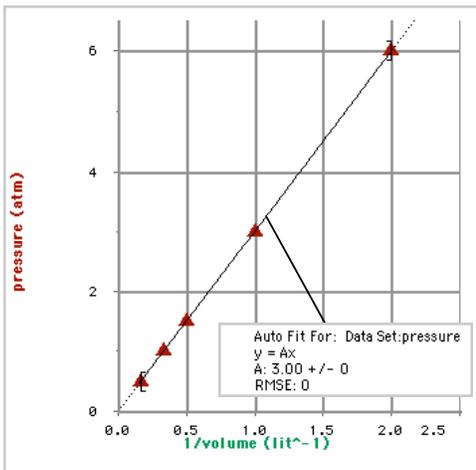
(continued)

Non-linear Relationships:



The graph to the left is a hyperbolic relationship. That is, the data are not linear and both sets of values appear to approach asymptotes. There are two approaches that one might use to find the relationship between the variables pressure and volume. One can perform a non-linear regression analysis to find the relationship using a calculator or computer, or linearize the plot by manipulating the data and re-plotting in an effort to create a linear relationship between the manipulated variables. Looking at the data carefully suggests that we have here an inverse relationship. That is, when pressure is doubled, the volume is halved. The relationship between the variables would seem to be of the form pressure is inversely proportional to volume. Assuming for a moment that this is the case, data for either pressure or volume can be inverted (e.g., replace by 1/var) and the data re-graphed to see if a linear relationship results. Replacing volume by 1/volume results in the graph below.

Linearized Plot of Above Data:



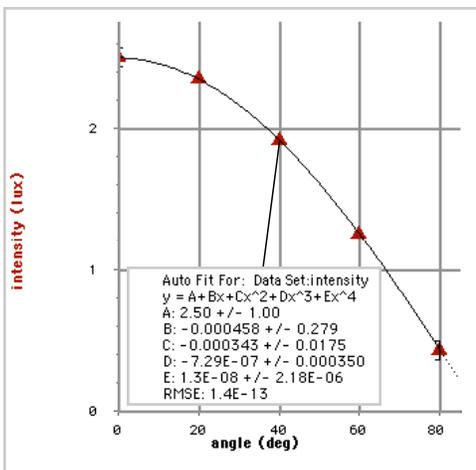
Here we see a linearized plot of the pressure-volume data used to create the graph above. On the horizontal axis of the graph the inverse of the volume has been plotted; pressure is plotted as before on the vertical axis. A linear fit of the type $y = mx + b$ is now possible and shows that a linear relationship exists between pressure and the inverse of the volume in this particular situation. Note that the x-variable is identified with 1/volume, that the y-variable is identified with 1/volume, and that $b = 0$. That is,

$$\text{pressure} = (3\text{atm} * \text{lit}) / \text{volume}$$

or

$$\text{pressure} * \text{volume} = 3\text{atm} * \text{lit}$$

The “Simpler, More Natural” Fit:



Students are sometimes tempted to use a polynomial function to conduct a non-linear curve fit using calculators or computers (e.g., $y = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + \dots$). Such a function will fit almost any data so long as the degree of the function is high enough. What sometimes happens though (as did not happen in the graph to the left) is that there will be wild migrations of the best-fit curve between the data points that probably wouldn't happen in nature. Fortunately, Mother Nature is less complex than that, and best fits involving trigonometric functions and the like can provide simpler forms of relationship. Note the 4th order power function fit of the graph to the left. A trigonometric function given by intensity = $(2.5\text{lux})\cos(\theta)$ is a “simpler” representation of the data and a “more natural” fit with the data. As a rule, simpler interpretations are valued over more complex in science.