Physics 220
Exam 2, Fall 2012

1) In a region of the planet Bovine, the gravitational acceleration near the surface is not constant as it is here, but in fact varies linearly with height so that the force of the Bovine gravity on an object of mass \( m \) is

\[
\vec{F}_g = -mgL \hat{y}
\]

Here we have chosen the + \( y \)-direction to be up and the \( x \)-direction to be parallel to the ground. The constant \( L \) is the scale length of the variation in the gravity and \( g \) is the standard gravitation acceleration. What is weirder still is that in addition to gravity, objects moving through the Bovine atmosphere only experience a quadratic drag force on their motion parallel to the ground, i.e. there is no drag for motion in the vertical direction.

\[
\vec{F}_{\text{drag}} = -cv_x^2 \hat{x}
\]

Thus the net force on an object moving through the Bovine atmosphere is

\[
\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_{\text{drag}} = -cv_x^2 \hat{x} - mgL \hat{y}
\]

The Black Angus Clan has developed a new extreme sport in which a bull runs off the edge of a cliff of height \( H \), moving with a speed \( v_0 \) horizontal to the ground. The goal of the sport is to travel as far in the \( x \)-direction as is possible.

a) Is the net force on the bull as it flies through the air conservative? Explain. (3 points)

b) Write a differential equation that describes the motion of the bull in the \( y \)-direction. (3 points)

c) Write the solution for the \( y \)-position of the bull as a function of time assuming that it starts from \( x = 0 \) and \( y = H \) at \( t = 0 \). (Hint: You can just write the general form down and use initial conditions to solve for the constants) (7 points)

d) At what time after launch, does the bull first hit the ground? (3 points)

e) Write and solve a differential equation that describes the velocity of the bull in the \( x \)-direction as a function of time again assuming that it starts from \( x = 0 \) and \( y = H \) at \( t = 0 \) (7 points)

f) Using the result of part e) find the position of the particle as a function of time. (7 points)

g) Determine the Range of the bull. (3 points)

2) The Holstein colony has figured out how to put a satellite into a low orbit around the planet. Assume that the period of the low orbit is \( T \) and the radius of the orbit is \( R \). They want to place the satellite into an orbit with a period 8 times longer. (Interestingly enough, the gravity away from the surface behaves like normal gravity.)

a) What is the radius of the new orbit in terms of \( R \)? (7 points)

b) If the speed of the satellite in the low orbit is \( V_0 \), by what factor would they have to increase the speed of the spacecraft in order to place it in a transfer ellipse that will take it out to the new orbit? (You do not need to calculate the second change in speed once it reaches the new orbit.) (7 points)

c) What is the eccentricity of the elliptical transfer orbit? (7 points)

d) For the elliptic transfer orbit, what is the velocity of the spacecraft at apogee (largest radius) in terms of the velocity at perigee (smallest radius)? (6 points)

e) What is the time required to go from the low orbit to the high orbit in terms of the period of the low orbit? (i.e. what is the transfer time?) (6 points)
3) Somewhere in space where gravity is negligible, the Hereford's are performing experiments (on their starship the BSS Elsie) about the nature of motion in an accelerated reference frame. In one particular experiment a small calf (who has a mass of 30 kg) is 4 m from the center of a merry-go-round that is rotating with constant angular velocity $\omega = 2$ rad/s and is walking radially outward with a speed of 3 m/s. During the experiment, the whole system is being accelerated upward with an acceleration of $A = 7$ m/s$^2$. Assume that the coefficient of friction between the calf and the merry-go-round is 0.6.

a) What is the magnitude and direction of the *coriolis* force on the calf? Clearly indicate this force on a diagram that includes the coordinates used and the direction of rotation. (8 points)
b) What is the magnitude and direction of the *centrifugal* force on the calf? Again, clearly indicate this force on a diagram that includes the coordinates used and the direction of rotation. (8 points)
c) Given that the coefficient of friction between the calf and the merry-go-round is 0.6, $\mu$ is the calf slipping (i.e. accelerating) in the rotating reference frame.
d) Assuming that the calf has the same position and velocity as above, what is the maximum value of $\omega$ so that the calf does not slip? (8 points)
a) **Dissipative Force ⇒ Non-Conservative**

b) \[ \ddot{y} = -\frac{mg}{L} y \]

\[ \ddot{y} + \frac{g}{L} y = 0 \quad \text{or} \quad \ddot{y} + \omega^2 y = 0 \quad \omega^2 = \frac{g}{L} \]

c) \[ y(t) = A \cos(\omega_0 t) + B \sin(\omega_0 t) \]

\[ \text{at } t = 0 \quad y(0) = H = A \]

\[ y(t) = H \cos(\omega_0 t) + B \sin(\omega_0 t) \]

\[ \dot{y}(t) = -\omega_0 H \sin(\omega_0 t) + \omega_0 B \cos(\omega_0 t) \]

\[ \text{at } t = 0 \quad \dot{y}(0) = 0 = \omega_0 B \Rightarrow B = 0 \]

\[ y(t) = H \cos(\omega_0 t) \]

d) **Hits ground \((y=0)\) when \(\cos(\omega_0 \tau) = 0\)**

\[ \omega_0 \tau = \frac{\pi}{2} \quad \Rightarrow \quad \tau = \frac{\pi}{2 \omega_0} \]

e) \[ m \ddot{x} = -c V_x^2 \]

\[ \frac{dv_x}{dt} = -\frac{c}{m} v_x^2 \quad \Rightarrow \quad \int_{v_0}^{v} \frac{dv_x}{v_x^2} = -\frac{c}{m} \int_{t_0}^{t} dt \]

\[ \frac{1}{v_x} \bigg|_{v_0}^{v} = -\frac{c}{m} t \]

\[ \frac{1}{v_0} - \frac{1}{v} = -\frac{c}{m} t \]

\[ \frac{1}{v} = \frac{1}{v_0} + \frac{c}{m} t = \frac{1}{v_0} \left(1 + \frac{cv_0}{m} t\right) \]

\[ v = \frac{v_0}{1 + \frac{cv_0}{m} t} \]
e) \[ \frac{dx}{dt} = \frac{V_0}{1 + \frac{cv_0}{m} t} \]

\[ \int_0^x dx = \int_0^t \frac{dt}{1 + \frac{cv_0}{m} t} \]

\[ X = V_0 \frac{m}{cv_0} \int_0^t \frac{ds}{s} \]

\[ X = \frac{m}{c} \ln (1 + \frac{cv_0}{m} t) \]

f) \[ R = (\sigma \omega) \]

\[ = \frac{m}{c} \ln \left[ 1 + \frac{cv_0}{m} \frac{\pi}{2 \omega_0} \right] \]

\[ R = \frac{m}{c} \ln \left[ 1 + \frac{\pi c v_0}{2m \sqrt{s}} \right] \]
2) From Kepler's 3rd Law

a) \[ T^2 = C R^3 \quad \Rightarrow \quad T_o^2 = C R_o^3 \]

\[
(8 T_o)^2 = C R_1^3
\]

\[ 64 (T_o^2) = 64 (C R_o^3) = C R_1^3 \]

\[ R_1 = (64)^{1/3} R_o = \sqrt[3]{4 R_o} = R_1 \]

b) From class Notes,

\[ \Delta V_1 = V_1 \left\{ \sqrt{\frac{2 R_o}{r_1 + r_2}} - 1 \right\} \]

\[ = V_0 \left\{ \frac{2 (4 R_o)}{R_o + 4 R_o} - 1 \right\} = V_0 \left\{ \frac{8 R_o}{5 R_o} - 1 \right\} \]

\[ \Delta V_1 = \frac{3}{5} V_0 \]

or, \[ V_{1e} = V_0 + \Delta V_1 = \frac{8}{5} V_0 \]

Must increase speed by 60%.

C) \[ r_{\text{min}} = R_o = \frac{\alpha}{1+e} \quad \Rightarrow \quad \alpha = (1+e) R_o \]

\[ r_{\text{max}} = 4 R_o = \frac{\alpha}{1-e} \quad \Rightarrow \quad \alpha = 4 (1-e) R_o \]

\[ R_o + 4 R_o = 4 R_o + 4 e R_o \]

\[ 5 e R_o = 3 R_o \]

\[ e = \frac{3}{5} \]
d) Use Conservation of $\mathbf{P}$ at these two points

$$R_0V_0 = 4R_0V_2$$

$$V_2 = \frac{1}{4} V_0$$

2) $T_{\text{trans}} = \frac{1}{2} T_{\text{ellipse}}$

$$T_{\text{ellipse}} = C (a^3) = C \left[ \left( \frac{R_0 + 4R_0}{2} \right) \right]^3$$

$$= C \left[ \left( \frac{5}{2} R_0 \right) \right]^3 = \left( \frac{5}{2} \right)^3 (C R_0^3)$$

$$= \left( \frac{5}{2} \right)^3 T_0^2$$

$$T_{\text{ellipse}} = \left( \frac{5}{2} \right)^{3/2} T_0$$

$$T_{\text{trans}} = \frac{1}{2} \left( \frac{5}{2} \right)^{3/2} T_0 = 1.98 T_0$$
3) $\vec{r}' = 4m \hat{e}_x$; $\vec{v}' = 3m/s \hat{e}_x$; $\vec{a} = 7m/s^2 \hat{e}_z$

$\omega = 2s^{-1} \hat{e}_z$; $m = 30kg$

\begin{align*}
a) \quad F_{\text{cor}} &= -2m(\omega \times v') = -2(30kg)(2s^{-1} \hat{e}_z \times 3m/s \hat{e}_x) \\
&= -360 \frac{kgs}{s^2} (\hat{e}_z \times \hat{e}_x) \\
\Rightarrow F_{\text{cor}} &= -360N \hat{e}_y
\end{align*}

\begin{align*}
b) \quad F_{\text{cent}} &= -m(\omega \times (\omega \times \hat{r}')) \\
&= -(30kg)(2s^{-1})(2s^{-1})(4m) \left[ \hat{e}_z \times (\hat{e}_z \times \hat{e}_x) \right] \\
&= -480N \left[ \hat{e}_z \times \hat{e}_x \right] \\
\Rightarrow F_{\text{cent}} &= +480N \hat{e}_x
\end{align*}

c) The upward accel. provides a normal force of $m\vec{a} = (30kg)(7m/s^2) = 210N$

$\Rightarrow$ Max Friction $= \mu N = (0.6)(210N) = 126N$

Friction is less than either force, $\Rightarrow$

cut must slide.
d) In order not to slip,

\[ m \cdot r^2 A > \sqrt{F_{tor}^2 + F_{cin}^2} \]

\[ m^2 A^2 > 4 \pi^2 V^2 A + m^2 \omega^2 r^2 \]

\[ \omega^4 r^2 + (4V^2) \omega^2 - m^2 A^2 = 0 \]

\[ \frac{\omega^4 + (4V^2) \omega^2 - m^2 A^2}{r^2} = 0 \]

\[ \omega^4 + 2.25 \omega^2 - 1.1025 = 0 \]

\[ \omega^2 = \frac{-2.25 \pm \sqrt{(2.25)^2 + 4 \cdot 1.1025}}{2} \]

\[ \omega^2 = \frac{-2.25 \pm \sqrt{5.0625}}{2} \]

\[ \omega^2 = \frac{-2.25 \pm 3.078}{2} \]

Only keep + sign since \( \omega^2 > 0 \)

\[ \omega^2 = 0.414 \]

\[ \omega = 0.64 \text{ rad/s} \]