Homework Set #5

1. Find the solution to the ordinary differential equation

\[ \frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + 4x = 0 \]

subject to the initial conditions that at \( t = 0 \), \( x = 5 \) and \( \frac{dx}{dt} = -3 \)

2. Find the general solution to the ordinary differential equation

\[ \frac{d^2x}{dt^2} + 7 \frac{dx}{dt} + 10x = 18t^2 \]

3. Find the general solution to the ordinary differential equation

\[ \frac{d^2x}{dt^2} + 4x = \sin(t) \sin(10t) \]

4. (F&C 3.5) A particle undergoing simple harmonic motion has a velocity \( \dot{x}_1 \) when the displacement is \( x_1 \) and a velocity \( \dot{x}_2 \) when the displacement is \( x_2 \). Find the angular frequency and the amplitude of the motion in terms of the given quantities.

5. A body of uniform cross-sectional area \( A \) and mass density \( \rho \) floats in a liquid and at equilibrium displaces a volume \( V \). Show that the period of small oscillations about the equilibrium position is given by

\[ \tau = 2\pi \sqrt{\frac{V}{gA}} \]

where \( g \) is the acceleration caused by gravity. (Hint: the two forces acting on the body are 1) gravity pulling it downwards and 2) the buoyant force (equal to the mass of the fluid displaced) pushing it upwards.)