Graphs and Units of Slope and Y-intercept

After students linearize a graph, they know that they can find the relationship between the two variables on the X- and Y-axes. They merely use the equation for a straight line \( y = mx + b \) where \( m \) is slope and \( b \) the Y-intercept. But what are the units of the slope and Y-intercept? One answer is that the units of the slope are the units of \( \Delta y / \Delta x \) and the units of the Y-intercept are the units of the Y-axis. The next best way to answer this question is to use information provided by the graphing program Logger Pro.

Say that students are given the data to the left for time \( t \) (expressed in seconds, s) and velocity \( v \) (expressed in meters per second, m/s). A graph is made plotting \((t, v)\) and a right-opening parabola results. Logger Pro’s “Data: New Calculated Column…” tool is used to square the velocity term. The data are then re-plotted \((t, v^2)\) and a linear relationship results as is shown in the graph below.

Carefully examine the linear fit for the data set involving \( v^2 \). (See the box within the graph.) Note carefully that the now linear relationship is given explicitly as \( v^2 = mt + b \).

Now, \( b \) (the Y-intercept) is given by Logger Pro as 76.00 m\(^2\)/s\(^2\). The physical form of the relationship is then properly written as follows, including units:

\[
v^2 = 347.5 \frac{m^2}{s^3} t + 76.00 \frac{m^2}{s^2}
\]

Note that when time is inserted into the equation (say \( t = 4.500s \)) the units work out properly for determining \( v \) which is expressed in m/s. That is,

\[
v^2 = 347.5 \frac{m^2}{s^3} (4.500s) + 76.00 \frac{m^2}{s^2}
\]

\[
v^2 = 1564. \frac{m^2}{s^2} + 76.00 \frac{m^2}{s^2}
\]

\[
v^2 = 1640. \frac{m^2}{s^2}
\]

\[
v = \sqrt{v^2} = \sqrt{1640. \frac{m^2}{s^2}} = 40.50 \frac{m}{s}
\]